MTH4100
Solutions to Exercise Sheet 10

1.:

$$\int_0^{\sqrt{\ln 2}} 2xe^{x^2}\,dx;\ \begin{bmatrix} u = x^2 \\ du = 2x\,dx \\ x = 0 \Rightarrow u = 0,\ x = \sqrt{\ln 2} \Rightarrow u = \ln 2 \end{bmatrix} \to \int_0^{\ln 2} e^u\,du = [e^u]_0^{\ln 2} = e^{\ln 2} - e^0 = 2 - 1 = 1$$

2.:

$$\int \frac{d\theta}{\sqrt{2\theta - \theta^2}} = \int \frac{d\theta}{\sqrt{1 - (\theta - 1)^2}};\ \begin{bmatrix} u = \theta - 1 \\ du = d\theta \end{bmatrix} \to \int \frac{du}{\sqrt{1 - u^2}} = \sin^{-1} u + C = \sin^{-1}(\theta - 1) + C$$

3.:

$$\int \cos^2 x\,dx = \int \frac{1}{2}(1 + \cos 2x)\,dx = \frac{x}{2} + \frac{\sin 2x}{4} + C$$

Alternatively via integration by parts:

$$\int \cos^2 x\,dx = \int (\sin x)' \cos x\,dx = \sin x \cos x + \int \sin^2 x\,dx = \sin x \cos x + \int (1 - \cos^2 x)\,dx$$

$$\Rightarrow \int \cos^2 x\,dx = \frac{1}{2}(x + \sin x \cos x) + C = \frac{x}{2} + \frac{\sin 2x}{4} + C$$

4.:

$$\int_{-\pi}^{0} \sqrt{1 - \cos^2\theta}\,d\theta = \int_{-\pi}^{0} |\sin\theta|\,d\theta;\ \begin{bmatrix} \sin\theta \le 0 \\ \text{for } -\pi \le \theta \le 0 \end{bmatrix} \to \int_{-\pi}^{0} -\sin\theta\,d\theta = [\cos\theta]_{-\pi}^{0} = \cos 0 - \cos(-\pi)$$
$$= 1 - (-1) = 2$$

5.:

$$\int_{\sqrt{2}}^{3} \frac{2x^3}{x^2 - 1}\,dx = \int_{\sqrt{2}}^{3} \left(2x + \frac{2x}{x^2 - 1}\right) dx = \left[x^2 + \ln|x^2 - 1|\right]_{\sqrt{2}}^{3} = (9 + \ln 8) - (2 + \ln 1) = 7 + \ln 8$$

6.:

$$\int \frac{1 - x}{\sqrt{1 - x^2}}\,dx = \int \frac{dx}{\sqrt{1 - x^2}} - \int \frac{x\,dx}{\sqrt{1 - x^2}} = \sin^{-1} x + \sqrt{1 - x^2} + C$$

7.:

$$\int \frac{dx}{1 + \sin x} = \int \frac{(1 - \sin x)}{(1 - \sin^2 x)}\,dx = \int \frac{(1 - \sin x)}{\cos^2 x}\,dx = \int (\sec^2 x - \sec x \tan x)\,dx = \tan x - \sec x + C$$