CW 3

1.) a) True: using the Fundamental Theorem of Calculus (FTC) Part 1, we have $h'(x) = f(x)$. f is differentiable for all x, hence h has a second derivative for all x ($h''(x) = f'(x)$).

b) True: h and h' are differentiable, hence they are continuous.

c) True: $h'(1) = f(1) = 0$

d) True: $h'(1) = f(1) = 0$ and $h''(1) = f'(1) < 0$

e) False: $h''(1) = f'(1) < 0$

f) False: $h''(1) = f'(1) < 0$ for all x, and hence never changes sign

g) True: $h'(1) = f(1) = 0$ and $h'(x) = f(x)$ is a decreasing function ($f'(x) < 0$).

2.) $I = \int \sqrt{1+\sin^2(x-1)}\ \sin(x-1)\cos(x-1)\,dx$

a) Let $u = x-1 \Rightarrow \dfrac{du}{dx} = 1 \Rightarrow du = dx$

$v = \sin u \Rightarrow dv = \cos u\, du$

$w = 1+v^2 \Rightarrow dw = 2v\, dv$

Hence $I = \int \sqrt{1+\sin^2 u}\ \sin u \cos u\, du$

$= \int \sqrt{1+v^2}\ v\, dv = \int \sqrt{w} \cdot \frac{1}{2}\, dw$

$= \frac{1}{2}\frac{2}{3} w^{\frac{3}{2}} + C = \frac{1}{3}(1+v^2)^{\frac{3}{2}} + C$

$= \frac{1}{3}(1+\sin^2 u)^{\frac{3}{2}} + C$

$= \frac{1}{3}\left(1+\sin^2(x-1)\right)^{\frac{3}{2}} + C$ //

b) Let $u = \sin(x-1) \Rightarrow du = \cos(x-1)\,dx$

$v = 1+u^2 \Rightarrow dv = 2u\, du$

get $I = \int \sqrt{1+u^2}\ u\, du = \frac{1}{2}\int \sqrt{v}\, dv$

$= \frac{1}{3} v^{\frac{3}{2}} + C = \frac{1}{3}(1+u^2)^{\frac{3}{2}} + C$

$= \frac{1}{3}\left(1+\sin^2(x-1)\right) + C$ //

c) Let $u = 1 + \sin^2(x-1)$

$$\Rightarrow \quad du = 2\sin(x-1)\cos(x-1)\,dx$$

get $\quad I = \int \frac{1}{2}\sqrt{u}\,du = \frac{1}{3}u^{\frac{3}{2}} + C$

$$= \frac{1}{3}\left(1 + \sin^2(x-1)\right)^{\frac{3}{2}} + C$$

3.) Let $\quad y = \int_u^{\frac{\pi}{6}} \cos(t^3)\,dt \quad$, with $u = \sqrt[3]{x}$

Then, using chain rule, get

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

$$= \left(\frac{d}{du}\int_u^{\frac{\pi}{6}} \cos(t^3)\,dt\right)\frac{du}{dx}$$

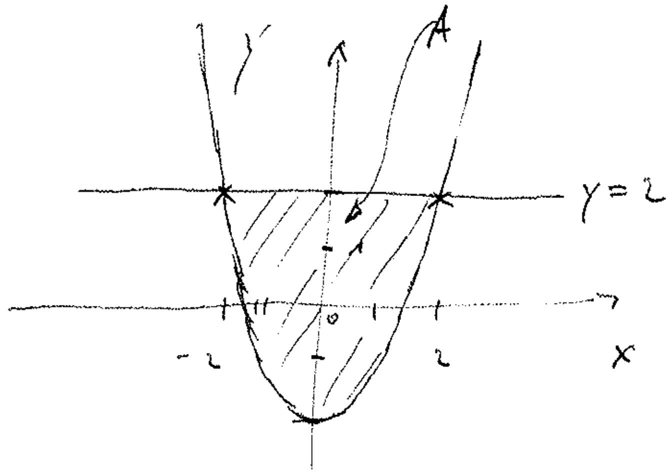
switching integration boundaries $\quad = -\left(\frac{d}{du}\int_{\frac{\pi}{6}}^{u} \cos(t^3)\,dt\right)\cdot\frac{1}{3}x^{-\frac{2}{3}}$

using FTC $\quad = -\cos(u^3)\cdot\frac{1}{3}x^{-\frac{2}{3}}$

$$= -\cos x \cdot \frac{1}{3}x^{-\frac{2}{3}}$$

$$= -\frac{1}{3}x^{-\frac{2}{3}}\cdot\cos x$$

4.) Area enclosed by $y = x^2 - 2$ and $y = 2$:



Points of intersection: $x^2 - 2 = 2$

$\Leftrightarrow x^2 = 4 \Rightarrow x = \pm 2$

$\Rightarrow$ graphs intersect at $(2, 2)$ and $(-2, 2)$

Also, on $[-2, 2]$: $2 \geq x^2 - 2$

Hence,

$$A = \int_{-2}^{2} \left[2 - (x^2 - 2)\right] dx$$

$$= \left[4x - \frac{1}{3}x^3\right]_{-2}^{2}$$

$$= \left(8 - \frac{8}{3}\right) - \left(-8 + \frac{8}{3}\right)$$

$$= \frac{32}{3}$$