<u>CW 8</u>

1.) a) $$\lim_{x\to 2} \frac{\sqrt{x^2+12}-4}{x-2} = \lim_{x\to 2} \frac{\frac{1}{2}(x^2+12)^{-\frac{1}{2}}\cdot 2x}{1}$$

$$= \frac{1}{2} \,//$$

(using l'Hôpital, as limit is of form "$\frac{0}{0}$")

b) $$\lim_{x\to 0} \frac{1-\cos 6x}{36x^2} = \lim_{x\to 0} \frac{+\cancel{6}\sin 6x}{\cancel{72}_{12}\, x}$$

$$= \lim_{x\to 0} \frac{6\cos 6x}{12} = \frac{1}{2} \,//$$

(using l'Hôpital twice, as limits are "$\frac{0}{0}$")

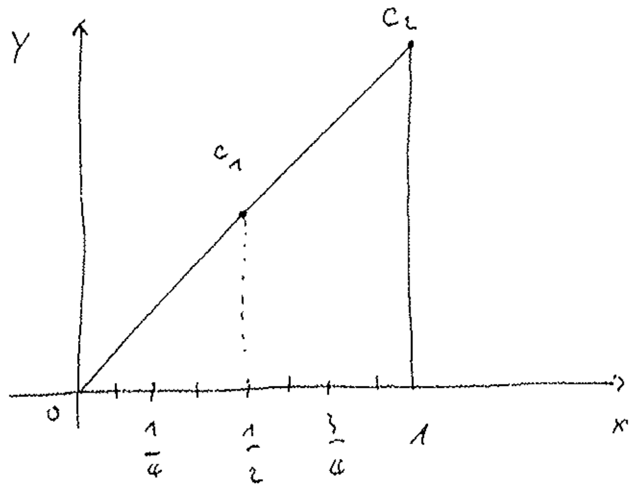
c) $$\lim_{x\to\infty} \frac{\sqrt{x+5}}{\sqrt{x}+5} = \lim_{x\to\infty} \frac{\sqrt{x}}{\sqrt{x}} \frac{\left(1+\frac{5}{x}\right)^{\frac{1}{2}}}{1+\frac{5}{\sqrt{x}}}$$

$$= \lim_{x\to\infty} \frac{\left(1+\frac{5}{x}\right)^{\frac{1}{2}}}{1+\frac{5}{\sqrt{x}}} = \frac{1}{1}$$

$$= 1 \,//$$

(l'Hôpital here not helpful)

2.) $f(x) = x$ over $[0, 1]$



i) $n = 2$: $\Delta x = \frac{b-a}{n} = \frac{1-0}{2} = \frac{1}{2}$

upper sum : $f(c_k)$ maximal, choose c_k at the right boundary of each subinterval (f monotonically increasing)

$c_1 = \frac{1}{2}$, $c_2 = 1$

$$A_n = \sum_{k=1}^{n} f(c_k)\,\Delta x$$

$$\Rightarrow \quad A_2 = f(c_1)\Delta x + f(c_2)\Delta x$$

$$= c_1 \Delta x + c_2 \Delta x = \frac{1}{2} \cdot \frac{1}{2} + 1 \cdot \frac{1}{2}$$

$$= \frac{3}{4}$$ //

(ii) $n=4 \qquad \Delta x = 1/4$

$c_1 = 1/4,\ c_2 = 1/2,\ c_3 = 3/4,\ c_4 = 1$

$$A_4 = \left(\tfrac{1}{4} + \tfrac{1}{2} + \tfrac{3}{4} + 1\right) 1/4 = 5/8$$

(iii) general $n \qquad \Delta x = 1/n$

$c_1 = 1/n,\ c_2 = 2/2,\ \ldots,\ c_k = k/n,\ \ldots\ c_n = 1$

$$A_n = \sum_{k=1}^{n} (k/n)\frac{1}{n} = \frac{1}{n^2}\sum_{k=1}^{n} k = \frac{1}{n^2} \times \frac{n(n+1)}{2}$$
$$= \frac{1}{2} + \frac{1}{2n}$$

Note that $\lim_{n\to\infty} A_n = 1/2 =$ Area or region

3 (a) $$\sum_{j=2}^{4} \frac{(-1)^{j-1}}{j-1} = -1 + \frac{1}{2} - \frac{1}{3} = -5/6$$

(b) $$\sum_{k=1}^{n} (k+2)(k-1) = \sum_{k=1}^{n}(k^2 + k - 2)$$
$$= \sum_{k=1}^{n} k^2 + \sum_{k=1}^{n} k - 2\sum_{k=1}^{n}$$
$$= \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} - 2n$$
$$= \frac{2n^3 + 6n^2 + 4n - 12n}{6}$$
$$= \frac{n^3 + 3n^2 - 4n}{3}$$