Strategy for Graphing $y = f(x)$

1. Identify the domain of f and any symmetries the curve may have.
2. Find y' and y''.
3. Find the critical points of f, and identify the function's behavior at each one.
4. Find where the curve is increasing and where it is decreasing.
5. Find the points of inflection, if any occur, and determine the concavity of the curve.
6. Identify any asymptotes.
7. Plot key points, such as the intercepts and the points found in Steps 3–5, and sketch the curve.

1.   [2007 exam question]

Sketch the graph of

$$f(x) = \frac{12}{3 + x^2}$$

by following step by step the strategy for graphing given above.

(*)2.   [2009 exam question]

Sketch the graph of

$$f(x) = -5x^2 + \frac{1}{4}x^4$$

by following step by step the strategy for graphing given above.

3.   [2009 exam question]

The sum of two non-negative real numbers is 20. Find the numbers if one number plus the square root of the other is to be as large as possible.