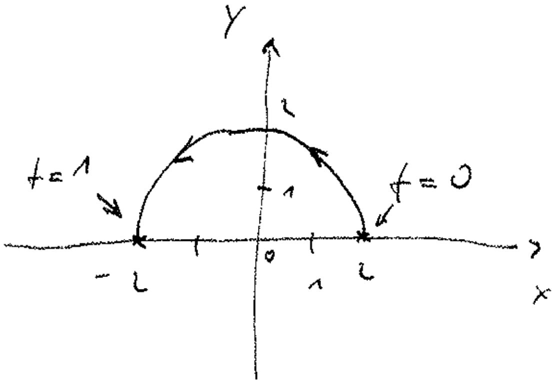
CW 6

1.) a) $x = 2\cos \pi t$, $y = 2 \sin \pi t$, $0 \leq t \leq 1$

$\Rightarrow \frac{x}{2} = \cos \pi t$, $\frac{y}{2} = \sin \pi t$, using $\cos^2\theta + \sin^2\theta = 1$

get $\frac{x^2}{4} + \frac{y^2}{4} = 1$ or $x^2 + y^2 = 4$

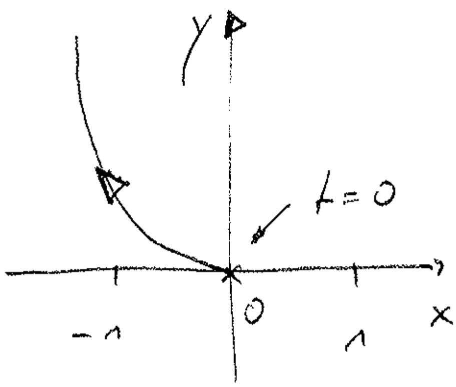
(circle, radius 2, centred at (0,0))



semi-circle in upper half-plane

b) $x = -\sqrt{t}$, $y = t$, $t \geq 0$

$\Rightarrow x = -\sqrt{y}$ or $y = x^2$, $x \leq 0$



parabola

2.) $x^3 + y^3 = 56 \quad (*)$

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $(-2, 4)$

Use implicit differentiation: differentiate both sides of $(*)$, get $3x^2 + 3y^2y' = 0$

$$\Rightarrow \quad y' = -\frac{x^2}{y^2}$$

and $6x + 3(2yy'y' + y^2y'') = 0$

$$\Rightarrow \quad 2x + 2y(y')^2 + y^2y'' = 0$$

$$\Rightarrow \quad y'' = -\frac{1}{y^2}\left(2x + 2y\, y'^2\right)$$

$$= -\frac{2}{y^2}\left(x + \frac{x^4}{y^3}\right)$$

Hence, at $(-2, 4)$ $\quad y' = -\frac{4}{16} = -\frac{1}{4}$ //

$$y'' = \frac{7}{32} \quad //$$

3.) The linearization of a differentiable function at point $x = x_0$ is

$$L(x) = f(x_0) + f'(x_0)(x - x_0)$$

Here: $x_0 = \frac{\pi}{2}$, $f(x) = \cos x$

$\Rightarrow$ $f'(x) = -\sin x$, $f(x_0) = 0$

$f'(x_0) = -1$

Get: $L(x) = 0 + (-1)\left(x - \frac{\pi}{2}\right)$

$= -x + \frac{\pi}{2}$ //

4.) $f_a(x) = 2x^3 + ax^2 + 1, \quad x, a \in \mathbb{R}$

a) Find critical points for each curve (keeping a fixed):

critical point $\Leftrightarrow$ interior point of domain of f where $f' = 0$ or f' undefined.

$\Rightarrow$ need $f_a' = 0$, get

$$6x^2 + 2ax = 0$$

or $x(6x + 2a) = 0 \quad \Rightarrow x = 0, \; x = -\frac{a}{3}$

$$f_a(0) = 1, \quad f_a\left(-\frac{a}{3}\right) = 2\left(-\frac{a}{3}\right)^3 + a\left(\frac{a}{3}\right)^2 + 1$$

$$= \frac{a^3}{27}(-2 + 3) + 1 = \frac{a^3}{27} + 1$$

Hence, critical points: $(0, 1)$ and $\left(-\frac{a}{3}, \frac{a^3}{27} + 1\right)$.

b) Use $\left(-\frac{a}{3}, \frac{a^3}{27} + 1\right)$, get $x = -\frac{a}{3}$ or $a = -3x$

and $y = \frac{a^3}{27} + 1 = -x^3 + 1$

$\Rightarrow$ Equation of curve: $y = 1 - x^3$