CW 5

1.) a) Def. of derivative:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \qquad (i)$$

or

$$f'(x) = \lim_{z \to x} \frac{f(z) - f(x)}{z - x} \qquad (ii)$$

b) Have to use (i):

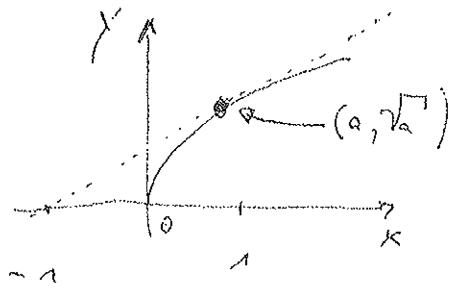
$$f'(x) = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$= \lim_{h \to 0} \frac{(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x})}{h \quad (\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{\cancel{x} + \cancel{h}^{1} - \cancel{x}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})}$$

$$= \frac{1}{2\sqrt{x}} \quad //$$

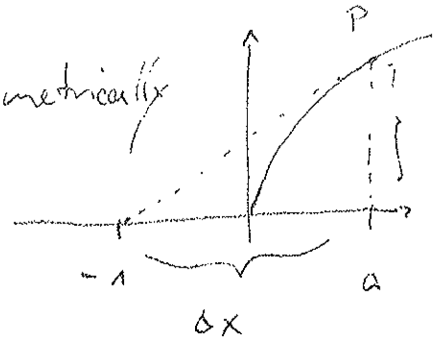
1.) c) Find tangent to $y = \sqrt{x}$ crossing x-axis at $x = -1$:



Let the tangent touch $y = \sqrt{x}$ at the point $(a, \sqrt{a})$.

Slope at $x = a$ is $f'(a) = \frac{1}{2\sqrt{a}}$

Also, geometrically



$$\frac{\Delta y}{\Delta x} = \frac{\sqrt{a}}{a-(-1)} = \frac{\sqrt{a}}{a+1}$$

The two expressions must be equal:

$$\frac{1}{2\sqrt{a}} = \frac{\sqrt{a}}{a+1} \quad \text{or} \quad a+1 = 2$$

$$\Rightarrow \quad a = 1$$

So point P where tangent meets the curve is $P = (x_0, y_0) = (1, 1)$

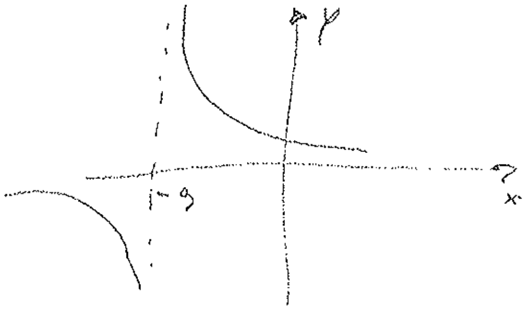
Hence, equation of tangent at $x_0 = a = 1$, $y_0 = \sqrt{a} = 1$:

$$y = y_0 + f'(x_0)(x - x_0)$$

$$= 1 + \frac{1}{2}(x - 1)$$

$$= \frac{1}{2}x + \frac{1}{2}$$

2.) $y = \dfrac{18}{x+9}$



Find lines with slope -2 :

$y' = -\dfrac{18}{(x+9)^2}$, for which x is $y' = -2$?

$\Rightarrow -2 = -\dfrac{18}{(x+9)^2} \Rightarrow (x+9)^2 = 9$

$\Rightarrow x+9 = \pm 3$, i.e. $x_1 = -12$
or $x_2 = -6$

Get two equations : $y = y_0 + y'(x - x_0)$

i) at $x = -12$, $y(-12) = -6$

$\Rightarrow y = -6 + (-2)(x+12)$
$= -2x - 30$ //

ii) at $x = -6$, $y(-6) = 6$

$\Rightarrow y = 6 - 2(x+6)$
$= -2x - 6$ //

3.) a) $y(x) = \dfrac{4x^5 + 8}{x^3}$ $\quad x \neq 0$

$$= 4x^2 + 8x^{-3}$$

$$y' = 8x - 24x^{-4}$$

$$y'' = 8 + 96x^{-5}$$

b) $q(t) = \tan\left(\dfrac{t}{\sqrt{t+2}}\right)$

i) $\tan x' = \left(\dfrac{\sin x}{\cos x}\right)' = \dfrac{\cos x \cdot \cos x - \sin x(-\sin x)}{(\cos x)^2}$

$$= \frac{1}{\cos^2 x}$$

ii) use chain rule:

$$q' = \frac{1}{\cos^2\left(\frac{t}{\sqrt{t+2}}\right)} \cdot \frac{1 \cdot \sqrt{t+2} - t \cdot \frac{1}{2}(t+2)^{-\frac{1}{2}}}{(\sqrt{t+2})^2}$$

$$= \frac{1}{\cos^2\left(\frac{t}{\sqrt{t+2}}\right)} \cdot \frac{t+2 - \frac{1}{2}t}{(t+2)^{\frac{3}{2}}}$$

$$= \frac{t+4}{2\cos^2\left(\frac{t}{\sqrt{t+2}}\right)(t+2)^{\frac{3}{2}}}$$

3.) e) $g(t) = \cos(2 - \sin 3t)$

$$g'(t) = -\sin(2-\sin 3t) \cdot \frac{d}{dt}(2 - \sin 3t)$$

$$= \sin(2 - \sin 3t) \cos 3t \cdot \frac{d}{dt}(3t)$$

$$= 3 \sin(2 - \sin 3t) \cdot \cos 3t$$

(using chain rule twice)