CW 4

1.) $$f(x) = \frac{-x^2+\alpha}{2x+4}$$

a) Find the asymptotes to $f(x)$.

i) Vertical asymptote: denominator zero

$\iff 2x+4=0 \iff x=-2$

ii) Horizontal or oblique asymptotes:

$\lim_{x\to\infty} f(x) = -\infty$, $\lim_{x\to-\infty} f(x) = +\infty$

$\Rightarrow$ no horizontal asymptotes.

However: oblique asymptote,

$$(-x^2+\alpha) : (2x+4) = -\tfrac{1}{2}x + 1 + \frac{\alpha-4}{2x+4}$$

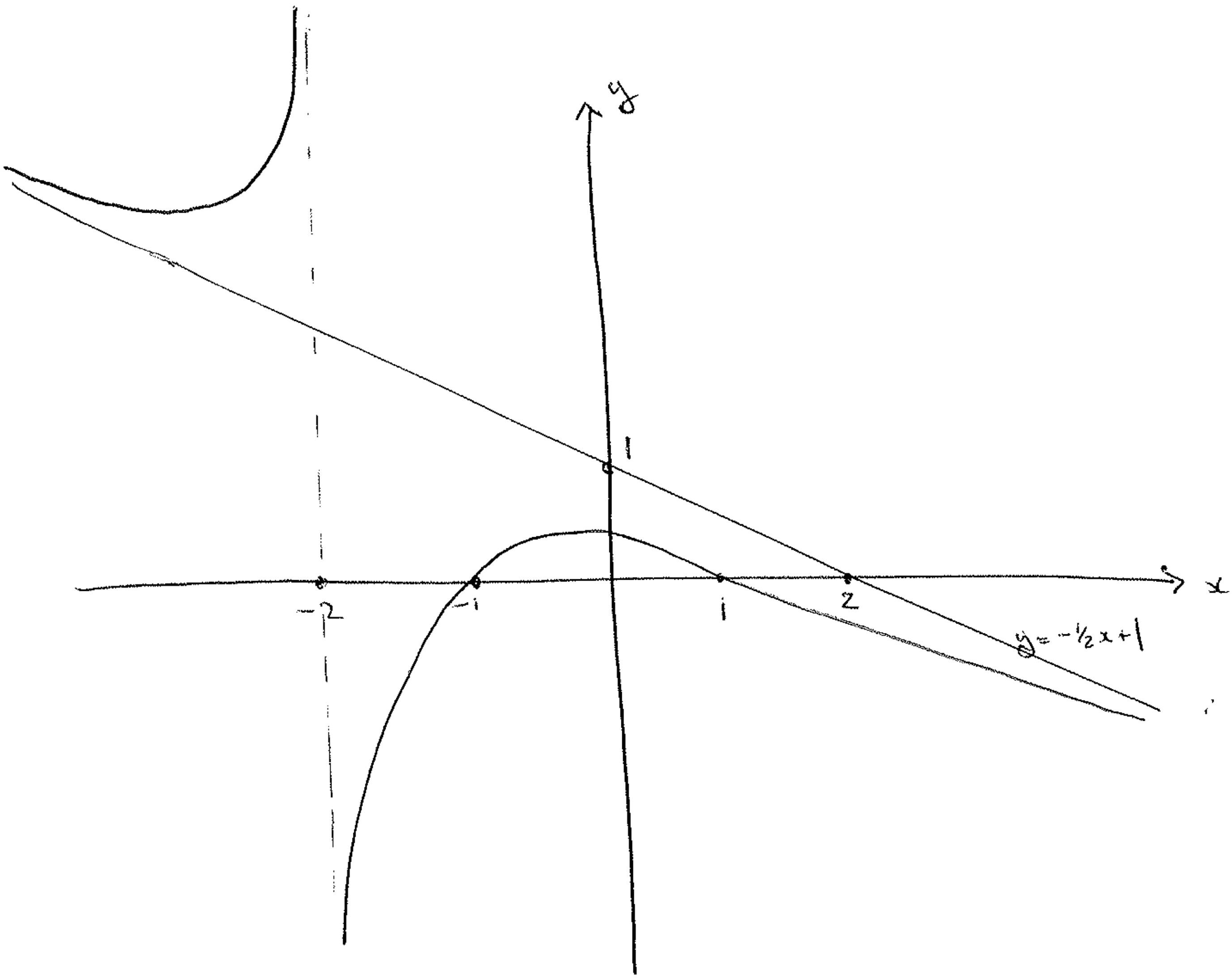
$\Rightarrow$ $y = -\frac{1}{2}x+1$ is the oblique asymptote to $f(x)$

b) Sketch the graph of $f(x)$ for $\alpha = 1$:

graph crosses x-axis at $-x^2+1=0$

or $x = \{1, -1\}$

1 (b) <u>Sketch</u>



2.) $f(x) = \frac{2x^2}{x-7}$

i) vertical asymptote : at $x = 7$

ii) oblique asymptote : $2x^2 : (x-7) = 2x + 14 + \frac{98}{x-7}$

$f(x) = 2x + 14 + \frac{98}{x-7}$

hence : $y = 2x + 14$ is the oblique asymptote

iii) horizontal asymptote :

as $\lim_{x \to \pm\infty} f(x) = \pm\infty$

there is no horizontal asymptote

3.) a) $g(x) = \frac{4x^2 - 100}{4x - 20} = \frac{x^2 - 25}{x - 5}$

for $x \neq 5$, $g(x) = \frac{(x+5)(x-5)}{x-5} = x+5$

Define $g(5) = 5 + 5 = 10$

$\Rightarrow \lim_{x \to 5} g(x) = g(5)$, extended $g(x)$ is continuous.

b) $f(x) = \begin{cases} x^2 - 1 & , \text{ for } x < 3 \\ 2ax & , \text{ for } x \geq 3 \end{cases}$

Need $f(x)$ to be continuous at $x = 3$.

Apply continuity test:

i) $f(3)$ exists

ii) $\lim_{x \to 3} f(x)$ exists

iii) $\lim_{x \to 3} f(x) = f(3)$

3.) b) start from ii)

$$\lim_{x \to 3^-} f(x) = 9 - 1 = 8$$

$$\lim_{x \to 3^+} f(x) = 2a3 = 6a$$

~~...~~ $\lim_{x \to 3} f(x)$ exists if $8 = 6a$

$$\Rightarrow a = \frac{4}{3}$$

check i) : $f(3) = 2 \cdot \frac{4}{3} \cdot 3 = 8$ exists

(for $a = \frac{4}{3}$)

check iii) : for $a = \frac{4}{3}$

$$\lim_{x \to 3} f(x) = 8 = f(3)$$

satisfied

$\Rightarrow$ $f(x)$ continuous for all x

($x^2 - 1$ and $\frac{8}{3}x$ are

continuous everywhere).

3.) c) $$f(x) = \frac{x(x^2-1)}{|x^2-1|}$$

denominator zero at $x = 1$ and $x = -1$

i) At $x = 1$:

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} \frac{x(x^2-1)}{|x^2-1|} = \lim_{x \to 1^-} \frac{x(x^2-1)}{-(x^2-1)}$$

$$= \lim_{x \to 1^-} (-x) = -1$$

$$\lim_{x \to 1^+} f(x) = \frac{x(x^2-1)}{(x^2-1)} = \lim_{x \to 1^+} x = 1$$

Hence, $\lim_{x \to 1} f(x)$ does not exist, f can not be extended to continuous function at $x = 1$.

ii) At $x = -1$:

$$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} \frac{x(x^2-1)}{|x^2-1|} = \lim_{x \to -1^-} \frac{x(x^2-1)}{(x^2-1)}$$

$$= \lim_{x \to -1^-} x = -1$$

3.) c) $\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} \frac{x(x^2-1)}{-(x^2-1)}$

$= \lim_{x \to -1^+} -x = 1$

$\Rightarrow \lim_{x \to -1} f(x)$ does not exist, function

can not be extended at $x = -1$

either.

Note: $+1^+ > +1 > +1^-$

and $-1^- < -1 < -1^+$

hence $\lim_{x \to -1^+} |x^2-1| = \lim_{x \to -1^+} -(x^2-1)$