

CW 3

$$1.) a) i) \lim_{x \rightarrow 1} \frac{x}{x-5} = \frac{1}{1-5} = -\frac{1}{4} //$$

$$ii) \lim_{x \rightarrow 5} \frac{x}{x-5} \text{ does not exist} //$$

$$\begin{aligned} b) \lim_{x \rightarrow 4} \frac{x^2 - 8x + 16}{x^2 - 5x + 4} &= \lim_{x \rightarrow 4} \frac{(x-4)^2}{(x-4)(x-1)} \\ &= \lim_{x \rightarrow 4} \frac{x-4}{x-1} = \frac{0}{3} = 0 // \end{aligned}$$

$$\begin{aligned} c) i) \lim_{x \rightarrow 5} \frac{x-5}{x^2 - 10x + 25} &= \lim_{x \rightarrow 5} \frac{x-5}{(x-5)^2} \\ &= \lim_{x \rightarrow 5} \frac{1}{x-5} \text{ does not exist} \end{aligned}$$

$$\begin{aligned} ii) \lim_{x \rightarrow 4} \frac{x-5}{x^2 - 10x + 5} &= \lim_{x \rightarrow 4} \frac{1}{x-5} \\ &= -1 // \end{aligned}$$

Note: In order to factor out e.g. $x^2 - 5x + 4$:
find zeroes, $x^2 - 5x + 4 = 0 \Rightarrow$

$$x_{1/2} = \frac{5 \pm \sqrt{25-16}}{2} = \frac{5 \pm 3}{2}, \quad x_1 = 4, \quad x_2 = 1$$

$$\text{Hence } x^2 - 5x + 4 = (x-4)(x-1)$$

$$2.) a) \lim_{x \rightarrow -3^-} (x+4) \frac{|x+3|}{x+3} = \lim_{x \rightarrow -3^-} (x+4) \frac{-(x+3)}{x+3}$$

$$= - \lim_{x \rightarrow -3^-} (x+4) = -1 //$$

$$b) \lim_{u \rightarrow 3} \frac{u^3 - 27}{u^4 - 81} = \lim_{u \rightarrow 3} \frac{(u-3)(u^2 + 3u + 9)}{(u-3)(u^3 + 3u^2 + 9u + 27)}$$

$$= \lim_{u \rightarrow 3} \frac{u^2 + 3u + 9}{u^3 + 3u^2 + 9u + 27} = \frac{3^2 + 3 \cdot 3 + 9}{3^3 + 3 \cdot 3^2 + 9 \cdot 3 + 27} = \frac{27}{4 \cdot 27}$$

$$= \frac{1}{4} //$$

$$c) \lim_{x \rightarrow 0} \frac{6x + 6x \cos(6x)}{\sin(6x) \cos(6x)} = \lim_{x \rightarrow 0} \frac{1 + \cos(6x)}{\left(\frac{\sin(6x)}{6x}\right) \cos(6x)}$$

$$= \frac{1 + \cos(0)}{1 \cdot \cos(0)} = 2 //$$

using
 $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

Note: as in 2b) : simple substitution
 gives: " $\frac{0}{0}$ " $\Rightarrow$ need to factor
 out terms.

$$3.) a) \lim_{t \rightarrow 0} \frac{\sin(4 - 4\cos(2t))}{1 - \cos(2t)} = 4 \lim_{t \rightarrow 0} \frac{\sin(4 - 4\cos(2t))}{4 - 4\cos(2t)}$$

$$= 4 \lim_{x \rightarrow 0} \frac{\sin x}{x} = 4 //$$

~~Note~~ Note:
 $(4 - 4\cos(2t)) \rightarrow 0$
as $t \rightarrow 0$

$$4) \lim_{t \rightarrow 5} \frac{t^2 + 3t - 40}{t^2 - 25} = \lim_{t \rightarrow 5} \frac{(t-5)(t+8)}{(t+5)(t-5)}$$

$$= \lim_{t \rightarrow 5} \frac{t+8}{t+5} = \frac{13}{10} //$$

$$c) \lim_{x \rightarrow 2} \frac{\sqrt{x^2+5} - 3}{x-2} = \lim_{x \rightarrow 2} \frac{\sqrt{x^2+5} - 3}{x-2} \cdot \frac{\sqrt{x^2+5} + 3}{\sqrt{x^2+5} + 3}$$

$$= \lim_{x \rightarrow 2} \frac{(x^2+5) - 9}{(x-2)(\sqrt{x^2+5} + 3)} = \lim_{x \rightarrow 2} \frac{x^2 - 4}{(x-2)(\sqrt{x^2+5} + 3)}$$

$$= \lim_{x \rightarrow 2} \frac{x+2}{\sqrt{x^2+5} + 3} = \frac{4}{6} = \frac{2}{3} //$$