Exercise Sheet 2

1.) a) $f(x) = 2 - x^2, \quad g(x) = \sqrt{x+2}$

domain: $D(f) = (-\infty, \infty)$

$D(g) = [-2, \infty)$

$$
\begin{aligned}
(f \circ g)(x) &= f(g(x)) \\
&= 2 - (g(x))^2 \\
&= 2 - \left(\sqrt{x+2}\right)^2 \\
&= -x
\end{aligned}
$$

$$
\begin{aligned}
(g \circ f)(x) &= g(f(x)) \\
&= \sqrt{f(x)+2} \\
&= \sqrt{2-x^2+2} \\
&= \sqrt{4-x^2}
\end{aligned}
$$

domain: $D(f\circ g) = [-2, \infty), \quad D(g \circ f) = [-2, 2]$

range: $R(f \circ g) = (-\infty, 2]$

$R(g \circ f) = [0, 2]$

1.) b) $f(x) = \sqrt{x}$, $g(x) = \sqrt{1-x}$

$D(f) = [0, \infty)$ $D(g) = (-\infty, 1]$

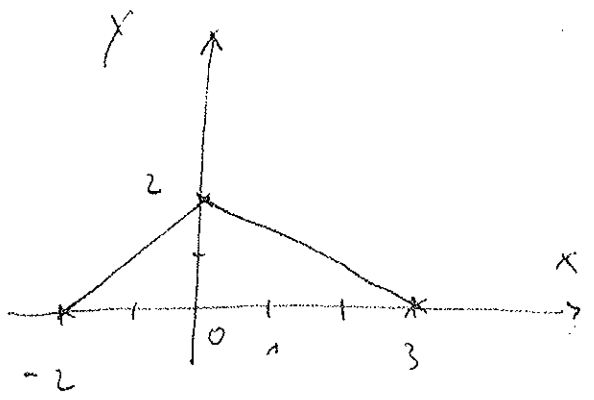
$$(f \circ g)(x) = f(g(x))$$
$$= \sqrt{g(x)}$$
$$= \sqrt{\sqrt{1-x}}$$
$$= \sqrt[4]{1-x} \; //$$

$$(g \circ f)(x) = g(f(x))$$
$$= \sqrt{1 - f(x)}$$
$$= \sqrt{1 - \sqrt{x}} \; //$$

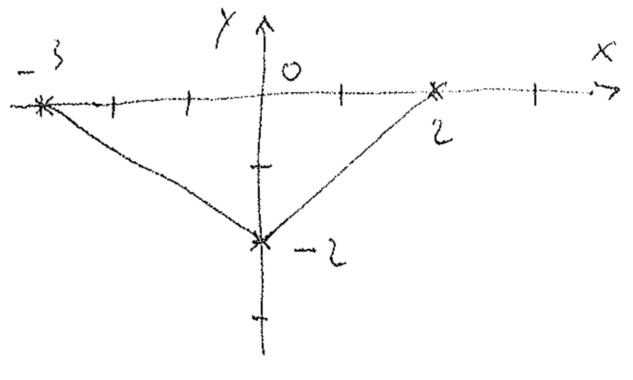
$D(f \circ g) = (-\infty, 1)$ $R(f \circ g) = [0, \infty)$

$D(g \circ f) = [0, 1]$ $R(g \circ f)$
$= [0, 1]$

2) a) $y = f(-x)$: graph reflected at y-axis

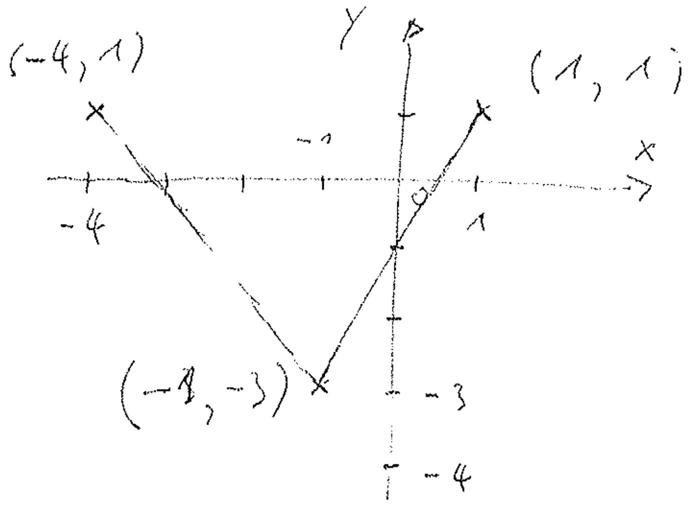


b) $y = -f(x)$: graph reflected at x-axis

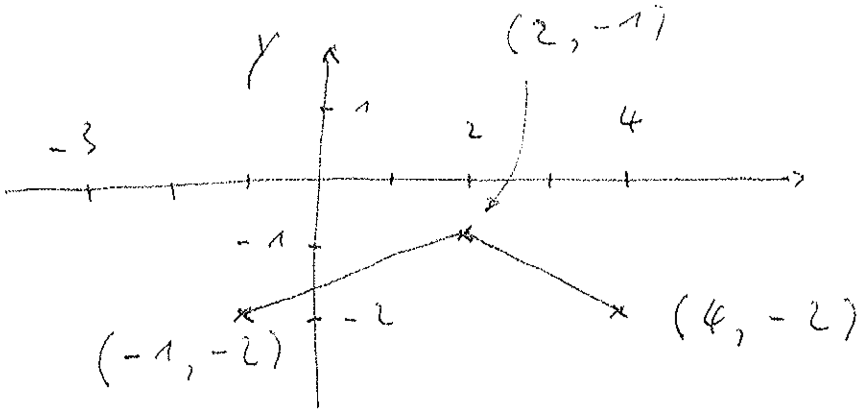


c) $y = -2\,f(x+1)+1$: graph shifted left by 1 unit, stretched vertically by a factor of 2, reflected about x-axis, then shifted upwards by 1 unit

2) c)



d) $y = \frac{1}{2} f(x-2) - 2$: graph is shifted right by 2 units, compressed by a factor of $\frac{1}{2}$, then shifted downwards by 2 units



3.) a) <u>Definitions</u>:

i) $f(x)$ even $\Leftrightarrow$ $f(-x) = f(x)$

ii) $f(x)$ odd $\Leftrightarrow$ $f(-x) = -f(x)$

b) $f(x) = 7x^5 - 3x^2$

$f(-x) = 7(-x)^5 - 3(-x)^2$

$= -7x^5 - 3x^2$

$\neq f(x)$ or $-f(x)$

Hence, $f(x)$ is neither even nor odd.

Exercise sheet 2

4) (i) $\cos^2\theta + \sin^2\theta = 1$

(ii) $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

(iii) $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

a) substitute $\theta = \alpha = \beta$ into (ii) and (iii), get

$$\cos 2\theta = \cos^2\theta - \sin^2\theta \qquad (iv)$$

$$\sin 2\theta = 2\sin\theta\cos\theta \qquad (v)$$

b) use (i) and (iv), get

$$\cos 2\theta = \cos^2\theta - (1 - \cos^2\theta)$$

$$\Leftrightarrow \quad \cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$$

and $\cos 2\theta = (1 - \sin^2\theta) - \sin^2\theta$

$$\Leftrightarrow \quad \sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$$

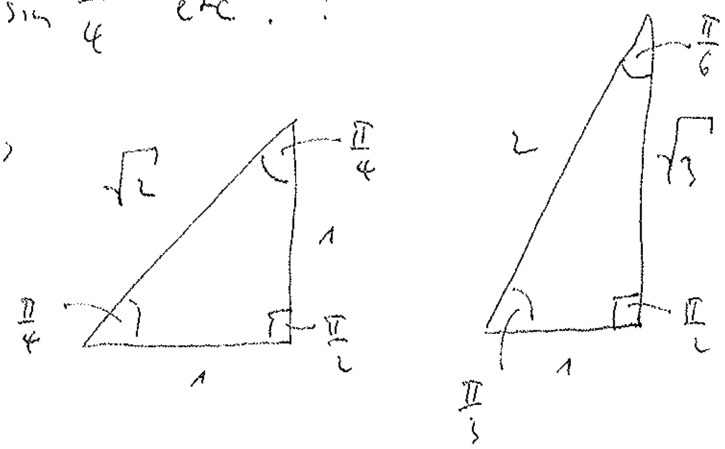
c) Rewrite $\frac{7}{12}\pi$ as $\frac{7}{12}\pi = \frac{\pi}{4} + \frac{\pi}{3}$

Now use (iii): $\sin\left(\frac{7}{12}\pi\right) = \sin\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$

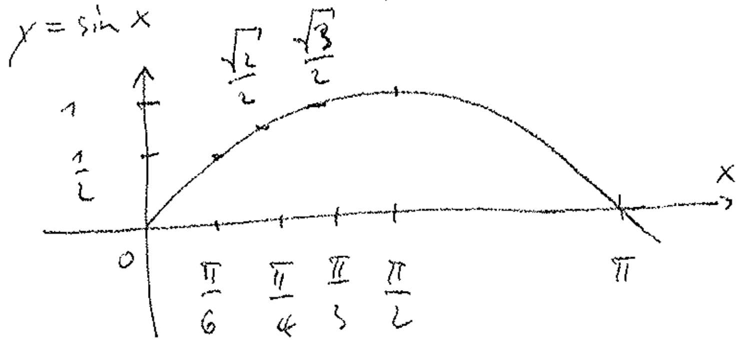
$$= \sin\frac{\pi}{4}\cos\frac{\pi}{3} + \cos\frac{\pi}{4}\sin\frac{\pi}{3}$$

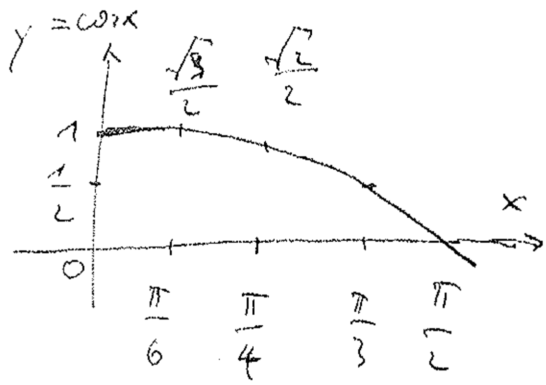
Need values for $\sin\frac{\pi}{4}$ etc.:

e.g. from triangles



or drawing the graph:





get: $\sin\left(\frac{7}{12}\pi\right) = \frac{\sqrt{2}}{2}\cdot\frac{1}{2} + \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2}$

$$= \frac{\sqrt{2}}{4}\left(1 + \sqrt{3}\right)$$

d) split $\frac{\pi}{12}$ into "simpler" values :

e.g. $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$

Now use (ii) :

$$\cos\left(\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$$

$$= \cos\frac{\pi}{3}\cos\left(-\frac{\pi}{4}\right) - \sin\frac{\pi}{3}\,\cancel{\cos}\,\sin\left(-\frac{\pi}{4}\right)$$

$$= \cos\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\sin\frac{\pi}{4}$$

$$= \frac{1}{2}\,\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\,\frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{2}}{4}\left(1 + \sqrt{3}\right)$$