MTH4100 Calculus 1, Autumn 2012

Exercise sheet 2 **Prof Bill Jackson**

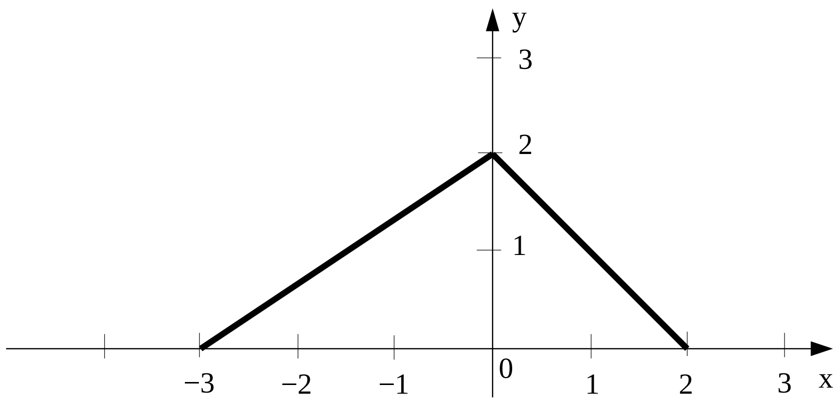
These questions are designed to help you understand the material covered in week $n, n \in \mathbb{N}$ lectures. Exercise sheets will typically be handed out in the Tuesday lecture of week $n+1$. You will get help on them in the exercise class on Tuesday or Wednesday of the same week. You should write up your solution to the starred question $(*)$ clearly and hand it in to your assigned helper during your week $n+2$ exercise class for feedback. Put your *full name and student number* on the top of your solution. It is important that you try to do ALL of the questions, not just the starred question.

(*)1. Find a formula for $f \circ g$ and $g \circ f$ and find the domain and range of each:

$$(a) \qquad f(x) = 2 - x^2 \,, \qquad g(x) = \sqrt{x+2}$$
$$(b) \qquad f(x) = \sqrt{x} \,, \qquad g(x) = \sqrt{1-x}$$

2. The graph of f is shown below. Draw the graph of each of the following functions:

(a) $y = f(-x)$, (b) $y = -f(x)$, (c) $y = -2f(x+1) + 1$, (d) $y = \frac{1}{2}f(x-2) - 2$.



[please turn over]

3. (a) Define what is meant by even and odd functions.

   (b) Then determine whether the function

$$f(x) = 7x^5 - 4x^2$$

   is even, odd, or neither. [2009 exam question]

4. Recall the identity

$$\cos^2\theta + \sin^2\theta = 1$$

   and the *addition formulas*

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta\,,\ \sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta\,,$$

   which are valid for all angles θ, α, β.

   (a) Derive the two *double-angle formulas*

$$\cos 2\theta = \cos^2\theta - \sin^2\theta\,,\ \sin 2\theta = 2\sin\theta\cos\theta$$

   by using (some of) the above three formulas.

   (b) Derive the two *half-angle formulas*

$$\cos^2\theta = (1+\cos 2\theta)/2\,,\ \sin^2\theta = (1-\cos 2\theta)/2$$

   by using (some of) the first three formulas and one of the formulas derived in 1.(a).

   (c) Use the above formulas to evaluate in terms of radicals $\sin\frac{7\pi}{12}$.

   (d) Evaluate in terms of radicals $\cos\frac{\pi}{12}$. [2007 exam question]