Queen Mary
University of London

B. Sc. Examination by course unit 2009

MTH4101 Calculus II

Duration: 2 hours

Date and time: 1 May 2009, 1430h–1630h

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

You should attempt all questions. Marks awarded are shown next to the questions.

Calculators are NOT permitted in this examination. The unauthorized use of a calculator constitutes an examination offence.

Complete all rough workings in the answer book and cross through any work which is not to be assessed.

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Exam papers must not be removed from the examination room.

Examiner(s): C. D. Murray

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Question 1 (a) If $z = x + iy$ is a complex number, find an expression for $|z - 1|$ in terms of x and y, and hence sketch the region in the Argand diagram for which $|z - 1| \leq 2$. [7]

(b) If $z_1 = 3(\cos 5\theta + i \sin 5\theta)$ and $z_2 = 2(\cos 3\theta + i \sin 3\theta)$ are two complex numbers, use de Moivre's theorem to simplify z_1^2/z_2^3. [7]

(c) Find the sum of the series

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{4^n}.$$

[7]

(d) Find the first four terms (i.e. up to and including terms of order x^9) of the binomial series of $(1 + x^3)^{-1/2}$. [7]

(e) Use the ratio test to find the radius and interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{x^n}{n\sqrt{n}\, 2^n}$$

being careful to specify the behaviour at the end points of the interval. [7]

(f) Find all first-order and second-order partial derivatives of the function $f(x, y) = e^y \cos x - e^x \sin y + 4x^2y^3 - 3 \ln x$. [7]

(g) Find the equation of the tangent plane and the equation of the normal line at the point $P = (2, 0, 2)$ on the surface consisting of those points (x, y, z) such that $2z - x^2 = 0$. [7]

(h) Find the Jacobian $\partial(x, y)/\partial(u, v)$ for the transformation $x = u \cos v$, $y = u \sin v$. [7]

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Question 2 (a) Use the comparison test to determine whether the following series converge or diverge: (i) $\sum_{n=4}^{\infty}(2/(n-3))$, (ii) $\sum_{n=1}^{\infty}(3/(n+1)^2)$. [6]

(b) An alternating series is given by $\sum_{n=0}^{\infty}(-1)^{n+1}u_n$. What conditions on u_n are sufficient for this series to converge? Hence determine whether the following series converges absolutely, converges conditionally or diverges:

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{n^3+2}\,.$$

[5]

Question 3 Use the method of Lagrange multipliers to find the maximum value of the function $f(x,y) = 36 - x^2 - y^2$ on the line $x + 3y = 10$. [11]

Question 4 Find all the local maxima, local minima and saddle points of the function

$$f(x,y) = 9x^3 + y^3/3 - 4xy\,.$$

[11]

Question 5 Solve the system $u = 2x + y$, $v = x - y$ to find expressions for x and y in terms of u and v. Use your solution to find the Jacobian $\partial(x,y)/\partial(u,v)$. Hence evaluate the integral

$$\int\int_R (2x^2 - xy - y^2)\ \mathrm{d}x\ \mathrm{d}y$$

for the region R bounded by the lines $y = -2x+1$, $y = -2x+3$, $y = x$, $y = x-1$. [11]

End of Paper

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