B. Sc. Examination by course unit 2011

MTH4100 Calculus 1

Duration: 2 hours

Date and time: 16 May 2011,   1000h–1200h

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

You should attempt all questions. Marks awarded are shown next to the questions.

Calculators are NOT permitted in this examination. The unauthorized use of a calculator constitutes an examination offence.

Complete all rough workings in the answer book and cross through any work which is not to be assessed.

Candidates should note that the Examination and Assessment Regulations state that possession of unauthorized materials by any candidate who is under examination conditions is an assessment offence. Please check your pockets now for any notes that you may have forgotten that are in your possession. If you have any, then please raise your hand and give them to an invigilator now.

Exam papers must not be removed from the examination room.

Examiner(s): Dr. K. Malik

Question 1   (a) Find the limit

$$\lim_{x \to 1^-} (2x + 1) \frac{|x - 1|}{x - 1} \ .$$

[5 marks]

(b) Find the limit

$$\lim_{t \to 4} \frac{t^2 - t - 12}{t^2 - 16} \ .$$

[5 marks]

(c) Given that

$$y^3 + x^3 + x = 6 \,,$$

and $y = y(x)$ find the values of dy/dx and d^2y/dx^2 at the point (-1,2).   [8 marks]

(d) Find the horizontal, vertical and oblique asymptotes, if any, of

$$f(x) = \frac{-8x^2}{2x - 1} \ .$$

[6 marks]

(e) Find the derivative $g'(x)$ of

$$g(x) = x^2 \cos\left(\sqrt{1 - x^2}\right) \quad, \quad |x| \le 1 \quad .$$

[6 marks]

(f) Find the area enclosed by the curves $y = -(x - 2)^2 + 4$ and $y = -x$.   [9 marks]

(g) Evaluate

$$\int 2xe^{x^2} \, dx \ .$$

[6 marks]

Question 2 Consider the curve $y = f(x)$ for the function

$$f(x) = -\frac{1}{2}x^4 + 4x^2 \ .$$

(a) Identify the domain of f and any symmetries the curve may have. [3 marks]

(b) Find $f'(x)$ and $f''(x)$. [2 marks]

(c) Find the critical points for f, and identify the function's behaviour at each one. [5 marks]

(d) Find where the curve is increasing and where it is decreasing. [3 marks]

(e) Find the inflection points for f, if any occur, and determine the concavity of the curve. [5 marks]

(f) Identify any asymptotes. [2 marks]

(g) Plot key points, such as intercepts, critical points, and points of inflection, and sketch the curve. [5 marks]

Question 3 (a) State both definitions of the derivative of the function $f(x)$ with respect to the variable x. [4 marks]

(b) Differentiate from first principles, $f(x) = \dfrac{1}{\sqrt{x}}$, that is, by using one of the above definitions of the derivative. [6 marks]

(c) State for which values f is continuous, and for which values f' is defined.[5 marks]

Question 4 Find all the minima and maxima of the function

$$f(x) = |2\sin(x)| - |\cos(2x)| \ ,$$

on the interval $I = [0, \pi]$. Indicate what type of minima and maxima (absolute or local) these are. [15 marks]

End of Paper