S357/J

The Open University

Third Level Course Examination 2003

Space, Time and Cosmology

Friday 17th October 2003 **10.00am–1.00pm**

Time allowed: 3 hours

This paper is divided into **TWO** parts, I and II. Part I carries 48 per cent of the total marks, and Part II carries 52 per cent. You are advised to spend roughly equal time on each part. Remember to allow time for reading the question paper first, and your answers afterwards. You will not be given extra time to do this.

You should answer **ALL** the questions in Part I, and **FOUR** questions from Part II.

Record your answers to Part I in the spaces provided in the green question and answer booklet inserted in this question paper. Answer **each** question attempted from Part II in a **separate** answer book (i.e. use **four** answer books in all). *It is most important that you indicate in the box provided on the front of each answer book the number of the question which you have answered in that book.*

At the end of the examination

1. Make sure that you have written your personal identifier and examination number on Part I of the question paper and on **each** answer book used. **Failure to do so will mean that your work cannot be identified.**
2. Complete the grid on the front of the enclosed green booklet containing Part I.
3. Put your signed desk record on top of Part I of the question paper and the answer books in which you have answered questions from Part II, and fix them together with the paper fastener provided.

Note: Part I of this paper is provided as a separate insert.

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PART I **AT THE END OF THE EXAMINATION, ATTACH THIS PART TO THE FRONT OF THE ANSWER BOOKS FOR PART II USING THE PAPER FASTENER PROVIDED.**

Examination No.								
Personal Identifier								

Please indicate on this grid which questions from Part II you have answered. (Details of Part I are *not* required.)

Part II			

Instructions for Part I

(*i*) Part I carries 48 per cent of the total marks, and you are advised to spend about **90 minutes** on it.

(*ii*) For each of the twelve questions, fill in the answer box(es) or space provided as indicated in the question.

(*iii*) Attempt as many questions as you can. There are **NO** penalties in Part I.

Question 1 (*4 marks*) Two particles, A and B, move under the influence of the same constant force $\mathbf{F}$ in an inertial frame S. At time t (in seconds) their position vectors (in metres) are

$$\mathbf{x}_A = (6t^2 + 3,\ -4t,\ t + 2), \quad \mathbf{x}_B = (3t^2 + 6t,\ -4,\ 4t - 1).$$

For each of (*a*) to (*d*), apply Newtonian mechanics and select **ONE** answer from the key for Q1.

(*a*) What is the ratio of the mass of B to that of A? ☐

(*b*) At what value of t (in seconds) do the particles collide? ☐

(*c*) What is the speed (in $\mathrm{m\,s^{-1}}$) of A relative to B when the particles collide? ☐

(*d*) Suppose that A and B combine to make a single particle, C. What is the magnitude (in $\mathrm{m\,s^{-2}}$) of the acceleration of C when subjected to $\mathbf{F}$ in frame S? ☐

KEY for Q1

A 1/5

B 1/4

C 1/3

D 1/2

E 1

F 2

G 3

H 4

I 5

Question 2 (*4 marks*) Observer O uses an inertial frame S.

Observer O_1 uses a frame with axes parallel to those of S, while its origin moves at constant velocity in S.

Observer O_2 uses a frame whose **1**-axis is identical to that of S at all times, while the other two axes are at a fixed angle relative to those of S.

Observer O_3 uses a frame whose **1**-axis is identical to that of S at all times, while the other two axes are rotating at constant angular speed relative to those of S.

Answer the following questions within the framework of Newtonian mechanics.

For each of (*a*) to (*d*), select **ONE** answer from the key for Q2.

(*a*) Who agrees with O about the distances between particles? ☐

(*b*) Who agrees with O about the kinetic energies of particles? ☐

(*c*) Who agrees with O about the momenta of particles? ☐

(*d*) Who agrees with O about the magnitudes of the acceleration of particles? ☐

KEY for Q2

A None of O_1, O_2, O_3

B Only O_1

C Only O_2

D Only O_3

E All but O_1

F All but O_2

G All but O_3

H All of O_1, O_2, O_3

Question 3 (*3 marks*) A hypothetical star S has two planets, A and B, which move in *circular* orbits, with radii R and $4R$, respectively.

For each of (a) to (c), determine the stated ratio and select **ONE** answer from the key for Q3.

(a) The ratio of the magnitude of acceleration of B to that of A. ☐

(b) The ratio of the period of the orbit of B to that of A. ☐

(c) The ratio of the rate at which the vector from S to B sweeps out area to the rate at which the vector from S to A does. ☐

KEY for Q3

A 1/16

B 1/8

C 1/4

D 1/2

E 1

F 2

G 4

H 8

I 16

Question 4 (*4 marks*) A *charged* particle is observed to move at a *constant* velocity $\mathbf{v}$ through a region containing constant uniform electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$, respectively. Let v, E and B denote the non-zero magnitudes of the vectors $\mathbf{v}$, $\mathbf{E}$ and $\mathbf{B}$, respectively.

Select from the key for Q4 the **THREE** conditions that **MUST** be satisfied in such a situation.

KEY for Q4

A $\mathbf{E} \cdot \mathbf{B} = 0$

B $\mathbf{v} \cdot \mathbf{E} = 0$

C $\mathbf{v} \cdot \mathbf{B} = 0$

D $v = B/E$

E $v = E/B$

F $v \geq B/E$

G $v \geq E/B$

H $v \leq B/E$

I $v \leq E/B$

Question 5 (*4 marks*) Figure 1 is a spacetime diagram drawn by an inertial observer in the spacetime of special relativity that has only one spatial dimension, x. Four lines, A, B, C, and D, and four events $\mathcal{E}_{\rm e}$, $\mathcal{E}_{\rm f}$, $\mathcal{E}_{\rm g}$ and $\mathcal{E}_{\rm h}$, are drawn in this diagram.

(*a*) Which of lines A–D can be the world-line of an electron? Select **ONE** answer from the key for Q5(*a*) and Q5(*b*).

(*b*) Which of lines A–D can be the world-line of a photon? Select **ONE** answer from the key for Q5(*a*) and Q5(*b*).

KEY for Q5(*a*) and Q5(*b*)

A Only line A

B Only line B

C Only line C

D Only line D

E Lines A and B

F Lines A and C

G Lines A and D

H Lines B and C

I Lines B and D

J Lines C and D

(*c*) Which, if any, of events $\mathcal{E}_{\rm e}$, $\mathcal{E}_{\rm f}$ and $\mathcal{E}_{\rm g}$ could be caused by event $\mathcal{E}_{\rm h}$? Select **ONE** answer from the key for Q5(*c*) and Q5(*d*).

(*d*) Which, if any, of events $\mathcal{E}_{\rm e}$, $\mathcal{E}_{\rm f}$ and $\mathcal{E}_{\rm g}$ could be judged to occur before event $\mathcal{E}_{\rm h}$, according to another appropriately chosen inertial observer? Select **ONE** answer from the key for Q5(*c*) and Q5(*d*).

KEY for Q5(*c*) and Q5(*d*)

A All of events $\mathcal{E}_{\rm e}$, $\mathcal{E}_{\rm f}$ and $\mathcal{E}_{\rm g}$

B Events $\mathcal{E}_{\rm e}$ and $\mathcal{E}_{\rm f}$

C Events $\mathcal{E}_{\rm e}$ and $\mathcal{E}_{\rm g}$

D Events $\mathcal{E}_{\rm f}$ and $\mathcal{E}_{\rm g}$

E Only event $\mathcal{E}_{\rm e}$

F Only event $\mathcal{E}_{\rm f}$

G Only event $\mathcal{E}_{\rm g}$

H None of events $\mathcal{E}_{\rm e}$, $\mathcal{E}_{\rm f}$ and $\mathcal{E}_{\rm g}$

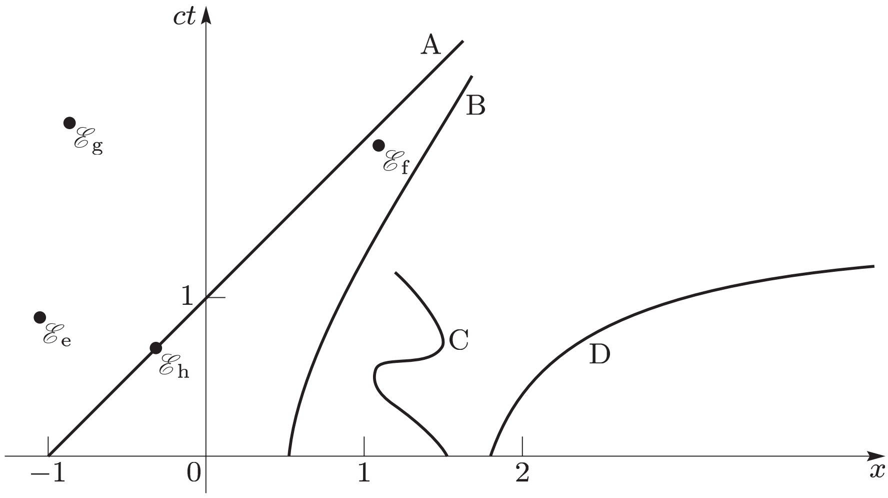

Figure 1 Spacetime diagram for Q5.

Question 6 (*4 marks*) In an inertial frame S, a particle A and an antiparticle B, each of mass m, move directly towards each other. Each has speed v. A head-on collision occurs, annihilating A and B and producing only a pair of photons.

In each part, select **ONE** answer from the key for Q6.

(a) What is the magnitude of the total relativistic momentum of the system before the collision? ☐

(b) What is the magnitude of the total relativistic momentum of the system after the collision? ☐

(c) What is the magnitude of the relativistic momentum of A, before the collision? ☐

(d) What is the magnitude of the relativistic momentum of one of the photons, after the collision? ☐

KEY for Q6

A 0

B mv

C mc

D $mv/\sqrt{1 - v^2/c^2}$

E $mc/\sqrt{1 - v^2/c^2}$

F $2mv$

G $2mc$

H $2mv/\sqrt{1 - v^2/c^2}$

I $2mc/\sqrt{1 - v^2/c^2}$

Question 7 (*5 marks*) Consider three very accurate identical clocks, A, B and C, which are synchronized at an event occurring at point P on the surface of a spherically symmetric non-rotating planet.

Clock A remains permanently at P.

Clock B is thrown vertically upwards and caught when it returns to P.

Clock C is sent on a journey along a closed path on the surface of the planet and returns to P.

When clocks A, B and C are reunited at P, their readings are found to be τ_A, τ_B and τ_C, respectively.

(a) Select from the key for Q7(a) the **THREE** relationships that are *true* within the framework of general relativity. [*Hint:* Clock B obeys the geodesic principle when it is in free fall.]

KEY for Q7(a)

A $\tau_A = \tau_B$

B $\tau_A > \tau_B$

C $\tau_A < \tau_B$

D $\tau_A = \tau_C$

E $\tau_A > \tau_C$

F $\tau_A < \tau_C$

G $\tau_B = \tau_C$

H $\tau_B > \tau_C$

I $\tau_B < \tau_C$

(b) Suppose that the magnitude of the downward acceleration due to gravity at the planet's surface is 10 m s^{-2} and that a tower of height 9 m is erected at P. An X-ray photon is emitted with energy E_1 at the top of the tower and received at the base P with energy E_2. Select from the key for Q7(b) the **ONE** item that is closest to $(E_2 - E_1)/E_1$.

KEY for Q7(b)

A 10^{-5}

B 10^{-10}

C 10^{-15}

D 10^{-20}

E 0

F -10^{-20}

G -10^{-15}

H -10^{-10}

I -10^{-5}

Question 8 (*4 marks*) Select from the key for Q8 the **FOUR** items that should be included in a concise but complete account of the field equations of general relativity.

KEY for Q8

A Energy density

B Ergosphere

C Geodesic principle

D Momentum density

E Momentum flux

F Principle of equivalence

G Ricci curvature

H Riemann curvature

Question 9 (*4 marks*) Select from the key for Q9 the **FOUR** statements about black holes that are believed to be *true*.

KEY for Q9

A A black hole can act as a gravitational lens.

B A black hole can have an observable charge.

C A black hole cannot be a source of electromagnetic radiation.

D Every black hole gives rise to a spherically symmetric metric.

E The spacetime curvature at the event horizon of a black hole is infinite.

F The locally measured speed of light goes to zero at the event horizon.

G Particles within the static limit may escape from a Kerr black hole.

H A black hole's spacetime singularity may be modified by quantum effects.

Question 10 (*4 marks*) The Robertson–Walker metric takes the form

$$(\Delta S)^2 = c^2(\Delta t)^2 - R^2(t)\left[\frac{(\Delta\sigma)^2}{1-k\sigma^2} + \sigma^2(\Delta\theta)^2 + \sigma^2\sin^2\theta(\Delta\phi)^2\right].$$

Select from the key for Q10 the **FOUR** statements that are *true*. | | | | |

KEY for Q10

A This form of metric violates the cosmological principle, since a cluster of galaxies near $\sigma = 0$ occupies a privileged position.

B The expansion of the Universe causes the comoving coordinate σ of a cluster of galaxies to increase.

C The evolution of $R(t)$ is determined solely by the pressure of radiation.

D If $k \leq 0$, then all values $\sigma \geq 0$ are allowed.

E If $k \leq 0$, then the derivative of $R(t)$ with respect to t cannot vanish at finite t.

F If $k = -1$, then the angles of a large triangle sum to less than 180°.

G If $k = 0$, then *spacetime* is flat.

H If $k = +1$, then the circumference of a circle with constant σ is less than $2\pi R(t)\sigma$.

Question 11 (*4 marks*) For each of (*a*) to (*d*), select **ONE** answer from the key for Q11 that describes how the given quantity depends on the scale factor R of the Robertson–Walker metric.

(*a*) The large-scale energy density of matter. ☐

(*b*) The temperature of the microwave background radiation. ☐

(*c*) The energy density of the microwave background radiation. ☐

(*d*) The pressure of the microwave background radiation. ☐

KEY for Q11

A Proportional to R^4

B Proportional to R^3

C Proportional to R^2

D Proportional to R

E Independent of R

F Proportional to $1/R$

G Proportional to $1/R^2$

H Proportional to $1/R^3$

I Proportional to $1/R^4$

Question 12 (*4 marks*) Fill in the blanks with appropriate letters from the key for Q12, using no term more than once.

The inflationary scenario seeks to solve a variety of cosmological puzzles, which include the following.

1) The problem of early homogeneity. This concerns the high degree of isotropy in the ☐. At the time of decoupling, regions with significantly different comoving coordinates could not have been in ☐, according to previous cosmological models.

2) The problem of close-to-critical density. This arises because the present energy density is fairly close to the critical value determined by the ☐. This implies that the ☐, k/R^2, was relatively unimportant at earlier epochs.

3) The problem of missing relics. Another perceived puzzle is that no-one has detected ☐, which some physicists believe may have been copiously produced in the early Universe.

A speculative solution is that quantum effects in the very early Universe induced a ☐ in the field equations of general relativity. This would result in ☐, which is called inflation. It is supposed that the inflationary epoch was followed by a process in which particles were created from ☐, liberated in a manner akin to the release of latent heat.

KEY for Q12

A Causal contact

B Cosmological constant

C Deceleration parameter

D Exponential expansion

E Hubble parameter

F Magnetic monopoles

G Microwave background radiation

H Curvature of spacetime

I Curvature of space

J Vacuum energy

PART II (*i*) *Part II carries 52 per cent of the total marks, and you are advised to spend about* **90 minutes** on it.

(*ii*) *Attempt* **FOUR** *questions only.*

(*iii*) *All answers carry thirteen marks, and where questions are divided into more than one section (labelled (a), (b), (c), etc.), the marks allocated to each section are indicated.*

(*iv*) *Write your answer to* **each** *question from Part II in a* **separate** *answer book. You should therefore submit* **FOUR** *answer books for this part.*

Question 13 An isolated system consists of three interacting non-relativistic particles: A, B and C. Suppose that A and B are *identical* and have mass m and that C has mass $\frac{1}{2}m$. In the inertial frame S of Figure 2, the initial positions and velocities are

$$\mathbf{x}_A(0) = (5,\ 0,\ 0) \qquad \mathbf{v}_A(0) = (0,\ 4,\ 0)$$
$$\mathbf{x}_B(0) = (-5,\ 0,\ 0) \qquad \mathbf{v}_B(0) = (0,\ -4,\ 0)$$
$$\mathbf{x}_C(0) = (0,\ 0,\ 9) \qquad \mathbf{v}_C(0) = (0,\ 0,\ 0)$$

with units of metres and seconds taken for distances and times respectively.

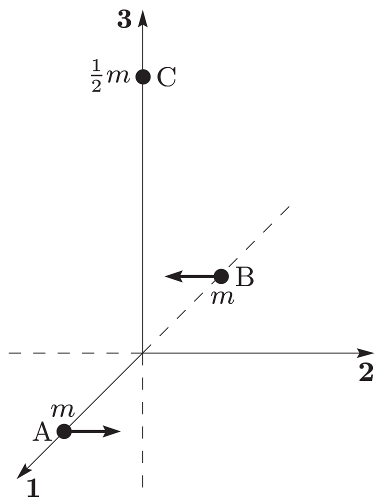

Figure 2 Initial configuration for Q13.

(*a*) Write down these initial conditions in frame S′ obtained by a rotation through 180° about the **3**-axis. (*2 marks*)

(*b*) "Suppose inertial observers in isolated frames S and S′ . . .". Complete the statement of the principle of relativity in Newtonian physics, making appropriate reference to identical particles and initial positions and velocities. (Any wording which clearly includes the essential points is acceptable.) (*2 marks*)

(*c*) Use your answer to sections (*a*) and (*b*) to explain why C cannot leave the **3**-axis. (*2 marks*)

(*d*) At a subsequent time t_1 it is found that

$$\mathbf{x}_A(t_1) = (0,\ -4,\ 3) \qquad \mathbf{v}_A(t_1) = (u,\ 0,\ w)$$
$$\mathbf{x}_C(t_1) = (0,\ 0,\ 2) \qquad \mathbf{v}_C(t_1) = (0,\ 0,\ -4)$$

Use your answers to sections (*a*) and (*b*) to write down $\mathbf{x}_B(t_1)$ and $\mathbf{v}_B(t_1)$. (*2 marks*)

(*e*) Use the conservation of linear momentum to find the value of w. (*2 marks*)

(*f*) Use the conservation of angular momentum to find the value of u and explain why energy conservation cannot be used for this purpose. (*3 marks*)

Question 14 This question concerns three events occurring in the spacetime of special relativity and described in an inertial frame with origin O.

Event $\mathcal{E}_0$ occurs at time $t = 0$, when two spaceships, A and B, depart simultaneously from O. Each travels at constant velocity in the same direction. Ship A has speed $0.6c$ and ship B has speed $0.8c$.

At event $\mathcal{E}_1$, ship B abruptly reverses its direction of motion and heads back to O with the same constant speed as before, while ship A maintains its constant velocity.

At event $\mathcal{E}_2$, the ships collide. This occurs at time $t = T$, according to an observer at rest at O.

(*a*) Sketch a spacetime diagram showing these three events, the world-lines of A and B between events $\mathcal{E}_0$ and $\mathcal{E}_2$, and the future light-cone of event $\mathcal{E}_0$. (Your sketch need not assign numerically precise values to the coordinates of $\mathcal{E}_1$ or $\mathcal{E}_2$.) (*4 marks*)

(*b*) (i) Use the speed of A to find the distance of event $\mathcal{E}_2$ from the origin.

(ii) Use the speed of B to find the distance of event $\mathcal{E}_1$ from the origin and the time at which it occurs. (*3 marks*)

(*c*) (i) Calculate the elapsed proper time between events $\mathcal{E}_0$ and $\mathcal{E}_2$ as recorded by a clock aboard A.

(ii) Calculate the elapsed proper time between events $\mathcal{E}_0$ and $\mathcal{E}_1$ as recorded by an identical clock aboard B.

(iii) Calculate the elapsed proper time between events $\mathcal{E}_1$ and $\mathcal{E}_2$ as recorded by the clock aboard B. (*3 marks*)

(*d*) Suppose that the captains of A and B were identical twins at event $\mathcal{E}_0$.

(i) Which twin is the younger when the ships collide?

(ii) Explain how the geodesic principle of general relativity is consistent with your finding. (*3 marks*)

Question 15 This questions concerns the Schwarzschild metric

$$(\Delta\tau)^2 = \left(1 - \frac{k}{r}\right)(\Delta t)^2 - \frac{1}{c^2}\left\{\frac{(\Delta r)^2}{1 - \dfrac{k}{r}} + r^2(\Delta\theta)^2 + r^2\sin^2\theta\,(\Delta\phi)^2\right\}$$

describing the spacetime exterior to a spherically symmetric black hole whose event horizon has radial coordinate k. The mass of the black hole is of the order of 10^9 solar masses.

A spaceship, with captain S, is maintained by its motors at fixed spatial coordinates $(r, \theta, \phi) = (3k/2, \pi/2, 0)$.

A distant observer, O, is located at fixed spatial coordinates $(R, \pi/2, 0)$, with $R >> 3k/2$.

Take care to explain how you interpret the metric when answering sections (a) and (b).

(a) Suppose that the spaceship emits a light signal radially outwards, with a frequency that S determines to be f_0.

(i) What frequency is detected by O?

(ii) When the signal leaves S, at what rate is its radial coordinate increasing according to O?

(iii) What is the corresponding rate according to S? (*5 marks*)

(b) Now suppose that S also emits a light signal tangentially. It is found to describe a circular orbit, at fixed radial coordinate $r = 3k/2$, in the plane with $\theta = \pi/2$.

(i) What is the angular speed of this orbit according to O?

(ii) What is the angular speed of this orbit according to S? (*4 marks*)

(c) Finally, suppose that S switches off her motors and commences free fall, radially inwards, while continuing to transmit radially outwards, towards O.

(i) Give a brief qualitative description of S's subsequent experiences.

(ii) Give a brief qualitative description of O's subsequent observations of S's signals. (*4 marks*)

Question 16 This question concerns the evolution of a homogeneous universe, for which Einstein's field equations lead to:

$$\left(\frac{1}{R(t)}\frac{\mathrm{d}R(t)}{\mathrm{d}t}\right)^2 + \frac{kc^2}{R^2(t)} = \frac{8\pi G}{3c^2}\rho(t)$$

where $R(t)$ and $\rho(t)$ are the scale factor and energy density at universal time t, k is the spatial curvature parameter, and

$$\frac{3c^2}{8\pi G} = 1.6 \times 10^{26}\,\mathrm{kg\,m^{-1}}.$$

(*a*) Suppose that at time t_0, the Hubble parameter is found to have the value $2.0 \times 10^{-18}\ \mathrm{s^{-1}}$. What value, $\rho_{\mathrm{crit}}(t_0)$, of the energy density at time t_0 will ensure that this universe is spatially flat? (*2 marks*)

(*b*) (i) State how the contributions of matter and radiation to the energy density depend on the scale factor.

(ii) Give brief explanations for the powers of $R(t)$ in your answers. (*4 marks*)

(*c*) Suppose that $\rho(t_0) > \rho_{\mathrm{crit}}(t_0)$.

(i) Make a rough sketch of $R(t)$ versus t.

(ii) Will a cosmic blueshift be observed for *all* distant clusters of galaxies if the Hubble parameter is negative at the time that the light is received? Explain your answer. (*4 marks*)

(*d*) Finally, suppose that $\rho(t_0) = \rho_{\mathrm{crit}}(t_0)$. Find the exponent n in the relationship $R(t) \propto t^n$ which agrees with the dependence of H on R at large t. (*3 marks*)

Question 17 Describe general relativity as a *geometric* theory of gravity. Besides topics of your own choice, your account should include:

(i) parallels with the geometry of two-dimensional surfaces;

(ii) an explanation of how the principle that describes the motions of the planets also describes the bending of starlight at an eclipse. (*13 marks*)

Aim to use fewer than about 300 words. Marks will be given primarily for the relevance and clarity of your explanation.

Question 18 Give a brief account of *three* main lines of evidence supporting the big bang theory of the early history of the Universe.

Besides topics of your own choice, your account should include:

(i) evidence that the Universe was much denser and hotter in the past;

(ii) evidence that it was highly isotropic. (*13 marks*)

Aim to use fewer than about 300 words. Marks will be given primarily for the relevance and clarity of your explanation.

[END OF QUESTION PAPER]