

### Question 5

Determine the least upper bound of the set

$$E = \left\{ 1 - \frac{2}{n^2} : n = 1, 2, \dots \right\}. \quad [4]$$

### Question 6

Determine whether or not each of the following sequences  $\{a_n\}$  converges. If it does converge, find its limit.

(a)  $a_n = \frac{n! + n^2}{3(n!) - n^3}$

(b)  $a_n = n!/6^n$  [6]

### Question 7

Determine whether or not each of the following series converges.

(a)  $\sum_{n=1}^{\infty} \frac{\sin(1/n)}{n^3}$

(b)  $\sum_{n=1}^{\infty} \frac{2^n}{n^3}$  [6]

### Question 8

- (a) The permutations  $f = (235)$ ,  $g = (13)(24)$ ,  $h = (13542)$  are elements of  $S_5$ . Write down each of the following as a permutation in cycle form:

$$f \circ g; \quad f \circ g \circ h; \quad h^{-1}; \quad f \circ g \circ f^{-1}.$$

- (b) Write down a group of permutations of elements of the set  $\{p, q, r, s\}$  which is isomorphic to the group  $(\{p, q, r, s\}, \circ)$  with the following Cayley table.

$\circ$	$p$	$q$	$r$	$s$
$p$	$s$	$r$	$q$	$p$
$q$	$r$	$s$	$p$	$q$
$r$	$q$	$p$	$s$	$r$
$s$	$p$	$q$	$r$	$s$

$$\begin{aligned} p &= (ps)(qr) \\ q &= (pr)(qs) \\ r &= (pq)(rs) \\ s &= (p)(q)(r)(s) \end{aligned}$$

[5]

### Question 9

Let  $\mathbb{R}^*$  be the group of non-zero real numbers under multiplication, and consider the function

$$\begin{aligned} \theta: \mathbb{R}^* &\longrightarrow \mathbb{R}^* \\ x &\longmapsto x^2. \end{aligned}$$

- (a) Prove that  $\theta$  is a homomorphism. ✓

- (b) Determine the kernel of  $\theta$ .  ~~$\{1, -1\}$~~

$$\{1, -1\}$$

$$\begin{aligned} \phi(x) \cdot \phi(y) &= \phi(x+y) \\ x^2 y^2 &= (x+y)^2 \end{aligned}$$

[4]