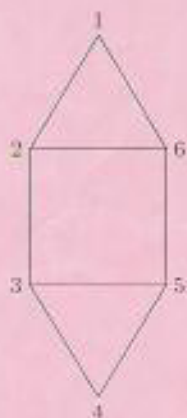


PART II

- (i) You should attempt no more than **THREE** questions from this part.
- (ii) Each question carries 10 marks. The mark allocation for each section of a question is given in square brackets beside the section.
- (iii) Start each question on a new page of your answer book.

Question 15

In this question, G is the symmetry group of the plane figure shown below. (It consists of a square with equilateral triangles attached to the upper and lower edges of the square.)



- (a) Write down the elements of G , in cycle notation, and the order of G . [2]
- (b) Find all the subgroups of G . [2]
- (c) Write down, with a brief reason, whether or not G is a cyclic group. [1]
- (d) Write down an isomorphism from G to one of the groups \mathbb{Z}_2 , \mathbb{Z}_4 , K_4 , $S(\triangle)$, $S(\square)$ or \mathbb{Z}_8 . [3]
- (e) Prove that G cannot be isomorphic to a subgroup of the symmetry group of the equilateral triangle. [2]

Question 16

Consider the following subset of \mathbb{R}^3 :

$$S = \{(a, b, -a - b) : a, b \in \mathbb{R}\}.$$

- (a) Show that S is a subspace of \mathbb{R}^3 . [2]
- (b) Prove that $\{(1, 1, -2), (-2, 1, 1)\}$ is a basis for S , and deduce the dimension of S . [4]
- (c) Find an orthogonal basis for S , which includes the vector $(1, 2, -3)$. [2]
- (d) Express the vector $(4, -1, -3)$ in terms of the basis in part (c). [2]