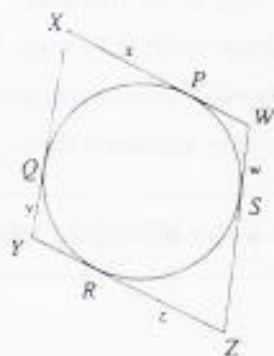


Question 19

(RESIT STUDENTS should attempt Question 19R on page 9 of this paper.)

- (a) The sides of the parallelogram $WXYZ$ are tangents to a circle at the points P , Q , R and S , and the lengths of the segments WP , XP , YQ and ZR are w , x , y and z respectively, as shown in the following diagram.



- (i) Show that

$$x + w = y + z$$

and

$$x + y = w + z.$$

- (ii) Deduce from the two equations in part (i) that

$$x = z$$

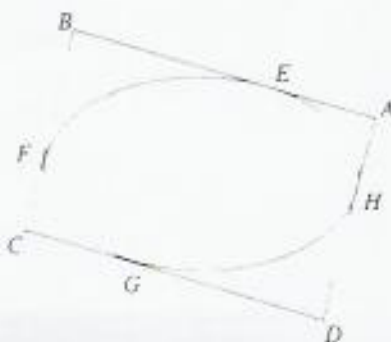
and

$$y = w.$$

- (iii) Deduce from the two equations in part (ii) that $WXYZ$ is a parallelogram with four sides of equal length.

[3]

- (b) The sides of the parallelogram $ABCD$ are tangents to an ellipse at the points E , F , G and H , as shown in the following diagram.



By using an affine transformation that maps the ellipse to a circle, together with the results of part (a), prove that

$$\frac{AE}{EB} = \frac{CF}{FD}$$

[4]