

Question 12

- (a) Find an affine transformation, t , of the plane, that sends the triangle with vertices whose position vectors are $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ to the triangle with vertices whose position vectors are $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$, $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$, preserving the order of the vertices given.
- (b) Find the inverse of t , giving your answer in the form

$$t^{-1} : \mathbf{x} \mapsto \mathbf{B}\mathbf{x} + \mathbf{b}, \quad [5]$$

Question 13

Prove that

$$\lim_{x \rightarrow 1} \frac{(1 + 3x)^{1/2} - (3 + 5x)^{1/3}}{x - 1}$$

exists, and determine its value.

[5]

Question 14

Calculate the Taylor polynomial $T_3(x)$ for the function $f(x) = x/(x+2)$ at -1 . Show that $T_3(x)$ approximates $f(x)$ to within $\frac{1}{8}$ on the interval $[-1, -\frac{1}{2}]$.

[5]