## PART I

(i) *You should attempt as many questions as you can in this part.*

(ii) *Write your answers in the answer book provided, beginning each question on a new page.*

(iii) *Questions in this part do not necessarily carry equal marks. The mark allocation is indicated for each question.*

### Question 1

The conic $x^2 - y^2 = 4$ is represented parametrically by the equations

$$x = t + \frac{1}{t}, \quad y = t - \frac{1}{t} \quad (t \neq 0).$$

(a) State whether this conic is an ellipse, a parabola, or a hyperbola. Sketch the curve, indicating on your sketch the points that correspond to the values $t = 1$ and $t = 2$ of the parameter $t$.

(b) Find the equations of the tangents to the conic at the points $t = 1$ and $t = 2$, and determine the co-ordinates of the point where they meet. [6]

### Question 2

(a) Determine the row-reduced form of the matrix

$$\begin{pmatrix} 1 & -2 & -1 & 2 \\ 3 & -5 & -5 & 7 \\ 2 & -8 & 6 & 0 \end{pmatrix}.$$

(b) Hence or otherwise, find the solution set of the equations

$$\begin{aligned} x - 2y - z &= 2, \\ 3x - 5y - 5z &= 7, \\ 2x - 8y + 6z &= 0. \end{aligned}$$

[5]

### Question 3

The set $S$ is defined by $S = \{(a, b, 2a - b) : a, b \in \mathbb{R}\}$.

(a) Show that $S$ is a subspace of $\mathbb{R}^3$.

(b) Show that the vectors $(1, 0, 2)$ and $(1, 1, 1)$ belong to $S$ and that the set $\{(1, 0, 2), (1, 1, 1)\}$ is a basis for $S$. Express the general vector $(a, b, 2a - b)$ as a linear combination of the vectors of this basis. [5]

### Question 4

Show that

$$\frac{7n}{n^2 + 12} < 1 \quad \text{for} \quad n > 4.$$

[4]

### Question 5

Determine whether each of the following sequences $\{a_n\}$ is convergent, stating the limit of the sequence (if it exists). You should name any result or test that you use.

(a) $a_n = \dfrac{4n^3 + 3n^2 + 2}{3n^3 + 2n}, \quad n = 1, 2, \ldots,$

(b) $a_n = \dfrac{4^n + 2(n!) + 3}{n^2 + 5^n}, \quad n = 1, 2, \ldots,$ [5]