Question 18

In this question M denotes the set of all 2×2 matrices and t is the function

$$t : (M, +) \longrightarrow (\mathbb{R}, +)$$
$$\begin{pmatrix} a & c \\ b & d \end{pmatrix} \longmapsto a + d.$$

(a) Prove that t is a group homomorphism and show that its kernel is H where $H = \left\{ \begin{pmatrix} a & c \\ b & -a \end{pmatrix} : a, b, c \in \mathbb{R} \right\}$. [3]

(b) Determine a typical element of the coset of H in M that contains the matrix $\begin{pmatrix} 2 & 0 \\ -1 & 4 \end{pmatrix}$. [2]

(c) Prove that t is onto. Hence, or otherwise, obtain an expression for a general coset of H in M. [3]

(d) Justify the statement that the quotient group M/H exists and identify this group up to isomorphism. [2]

Question 19

Note that the two parts of this question refer to two different quadric surfaces.

(a) Find a translation to reduce the quadric surface

$$2x^2 - y^2 - 3z^2 - 8x - 2y = 2$$

to standard form, and hence classify the quadric surface. [3]

(b) Reduce the equation of the quadric surface

$$z^2 - xy - 1 = 0$$

to standard form, and hence classify the quadric surface. [7]

Question 20

This question concerns the linear flow whose velocity function is

$$V(x, y) = (2x + 3y, 4x + 3y).$$

(a) Write down

(i) the matrix A of the flow;

(ii) the first order differential equations satisfied by the co-ordinate functions f and g of any flow function $\alpha = (f, g)$ for this flow;

(iii) a second order differential equation satisfied by both f and g. [3]

(b) Find the general solution of the differential equation in part (a)(iii). [3]

(c) Determine the flow function α for V that satisfies $\alpha(0) = (5, 2)$. [4]

[END OF QUESTION PAPER]