

PART I

- (i) You should attempt as many questions as you can in this part.
- (ii) Write your answers in the answer book provided, beginning each question on a new page.
- (iii) Questions in this part do not necessarily carry equal marks. The mark allocation is indicated in square brackets beside each question.

Question 1

Draw a sketch of the graph of the function f defined by

$$f(x) = \frac{4x+6}{x-3}.$$

Your sketch should include:

- (a) any asymptotes for the graph;
- (b) any points where the graph crosses the axes.

[4]

Question 2

The position vectors of points a and b are $\mathbf{a} = (1, 2)$ and $\mathbf{b} = (4, 3)$, respectively.

- (a) Draw a sketch showing the points a and b in the plane, and the line ℓ passing through a and b .
- (b) Find the position vector \mathbf{r} of a general point on the line ℓ .
 $\mathbf{r} = (1, 2) + \lambda(3, 1)$
- (c) Find the point p on ℓ whose position vector is perpendicular to ℓ . Mark p on your sketch.

[5]

Question 3

Find the matrix of the linear transformation

$$\begin{aligned} f: \mathbb{R}^2 &\longrightarrow \mathbb{R}^2 \\ (x, y) &\longmapsto (2x - y, 3x + y) \end{aligned}$$

with respect to

- (a) the standard basis in both domain and codomain;
- (b) the basis $\{(1, 2), (-2, 1)\}$ in the domain and the standard basis in the codomain;
- (c) the basis $\{(1, 2), (-2, 1)\}$ in both the domain and the codomain.

[5]

Question 4

The vectors \mathbf{e}_1 , \mathbf{e}_2 and \mathbf{e}_3 are defined by

$$\mathbf{e}_1 = \left(\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}\right), \mathbf{e}_2 = (2, -2, -3) \text{ and } \mathbf{e}_3 = (4, 0, 1).$$

- (a) Show that \mathbf{e}_1 and \mathbf{e}_2 are orthogonal.
- (b) Find the cosine of the angle between \mathbf{e}_2 and \mathbf{e}_3 .
- (c) Find a vector \mathbf{e}_4 that is orthogonal to \mathbf{e}_1 and \mathbf{e}_2 .
- (d) Find an orthonormal basis for \mathbb{R}^3 that includes the vector \mathbf{e}_1 .

[6]