

Question 5

Solve the inequality

$$\frac{4}{x} < \frac{x}{x-1}. \quad [5]$$

Question 6

Determine whether each of the following sequences $\{a_n\}$ is convergent, stating the limit of the sequence (if it exists). You should name any result or test that you use.

(a) $a_n = \frac{n^2 + 4}{3n + 1}, \quad n = 1, 2, \dots$

(b) $a_n = \frac{3^n - n! - 3}{3(n! + n + 1)}, \quad n = 1, 2, \dots \quad [5]$

Question 7

This question is about the symmetry group G of the square shown below.



Let $g \in G$ be the anti-clockwise rotation through $\pi/2$ about the centre, and let $h \in G$ be the reflection in the line through 1 and 3.

- Write g , g^2 and h in cycle form, using the numbering of the locations of the vertices as shown.
- Express the conjugate hgh^{-1} of g by h in cycle form and state its order.
- Give a brief reason why g^2 and h are not conjugate in G . [5]

Question 8

Let $\phi: (\mathbb{Z}_8, +_8) \rightarrow (\mathbb{Z}_5^*, \times_5)$ be the homomorphism given by $\phi(u) = 2^u \pmod{5}$.

- Find the image under ϕ of each element of \mathbb{Z}_8 .
- Find $\text{Ker}(\phi)$ and $\text{Im}(\phi)$ and the cosets of $\text{Ker}(\phi)$ in \mathbb{Z}_8 . [5]

Question 9

Sketch, on three separate diagrams, each of the following Circles together with its image under inversion in the unit circle.

- S_1 : the circle with centre $(\frac{1}{2}, 0)$ and radius $\frac{1}{2}$;
- S_2 : the line $x = \frac{1}{2}$;
- S_3 : the line $x = 2$. [4]

You should indicate clearly on each diagram the Circle and its image.