

### Question 10

- (i) Let  $h$  be the function

$$\text{Pr}[\text{Cn}[s, s], \text{Cn}[\text{dif}, \text{Cn}[s, \text{id}_3^3], \text{id}_2^3]] ,$$

where  $\text{dif}$  is the function defined by  $\text{dif}(x, y) = \begin{cases} x - y, & \text{if } x \geq y, \\ 0, & \text{if } x < y. \end{cases}$

Compute the values of

(a)  $h(3, 0)$ ,

(b)  $h(3, 2)$ .

[4]

- (ii) In this part you may present your arguments using either formal or informal definitions.

- (a) Show that the product function is primitive recursive by defining it in terms of the initial functions.

[3]

- (b) Show that the function

$$f(x_1, x_2, x_3) = x_2 \cdot x_1^{x_3}$$

is primitive recursive by defining it in terms of the initial functions and, if you wish, the sum and product functions.

[2]

- (iii) Write down the pairs  $(x_1, x_2)$  of natural numbers for which  $\text{Mn}[f](x_1, x_2)$  is defined, where  $f$  is the function defined by

$$f(x_1, x_2, y) = (x_2 \div (x_1 + y)) + (8 \div x_1) \cdot y.$$

[2]

### Question 11

- (i) A Turing machine has a configuration with left number 55 and right number 26. Draw the configuration which results when the scanning head has moved one square to the left, and write down the new left and right numbers.

[3]

- (ii) In parts (a), (b) and (c) below, you may use any of the recursive functions except  $\text{Eq}$ , or results about recursive functions, given in the Logic Handbook without proving that they are recursive. You may give your answers as informal definitions.

- (a) Show that the condition " $x = y$ " on pairs  $(x, y)$  of natural numbers is primitive recursive.

[2½]

- (b) Prove that if  $C_1$  and  $C_2$  are both primitive recursive conditions on pairs  $(x, y)$  of natural numbers, then so is the condition " $C_1$  and  $C_2$ ".

[2]

- (c) Show that the function  $f$  defined by

$$f(x, y) = \begin{cases} 3xy, & \text{if } 5x + y \text{ is even,} \\ y^4, & \text{if } x \text{ is odd and } 2x = y, \\ 5, & \text{otherwise,} \end{cases}$$

is primitive recursive.

[3½]