

### Question 15

For each of the following sentences, decide whether or not it is a theorem of  $Q$ . If it is a theorem of  $Q$ , write down a formal proof showing this. If it is not a theorem of  $Q$ , justify this. (You may use without proof the fact that all the axioms of  $Q$  are true under the interpretations  $N^*$  and  $N^{**}$  given in the Logic Handbook.)

(i)  $\forall x((0 \cdot x) + (x \cdot 0)) = ((0 + 0) \cdot x)$

(ii)  $\exists x \forall y (y' \cdot x) = x$

(iii)  $\exists y \forall x (x \cdot y') = x$

[11]

### Question 16

(i) Explain briefly what is meant by saying that the theory  $Z$  (of Elementary Peano Arithmetic) is consistent.

[2]

(ii) (a) Give a formal proof in the theory  $Z$  to show that

$$\vdash_Z -0 = 1.$$

[2]

(b) Deduce that if  $Z$  is consistent then  $0 = 1$  is not a theorem of  $Z$ .

[1]

(iii) Explain briefly when the formula  $\text{Prov}(y)$  is true in the standard interpretation.

[2]

(iv) Which theorem(s) of the course give(s) an answer to Hilbert's Question:

*Can the consistency of number theory be proved using only non-dubious principles of finitary reasoning?*

Explain why the theorem(s) answer(s) the question.

(Your answer may include references to any of the theorems listed in the Logic Handbook.)

[4]

[END OF QUESTION PAPER]