Question 13

(i) For each of the following conics (which you may assume are non-degenerate) locate the branch points when they exist.

(a) $2z^2 + 2zw + w^2 - z - 3w + 3 = 0$; [4]

(b) $4z^2 + 4zw + w^2 - z - 2w + 3 = 0$; [3]

(ii) Hence categorize each of them as one of the following:

(a) homeomorphic to the z-sphere;

(b) a double cover of the z-sphere branched over two points;

(c) a double cover of the z-sphere branched over two points and having a pinch point. [4]

Question 14

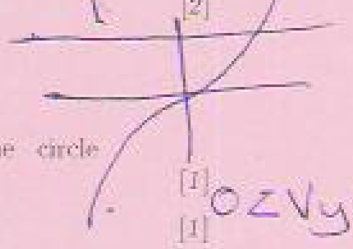
In this question we consider unbranched two-fold coverings of $k\mathbb{RP}^2$ by m fold tori, $m\mathbf{T}^2$, for suitable values of k and m.

(i) For what value of k can there be an unbranched covering $2\mathbf{T}^2 \longrightarrow k\mathbb{RP}^2$? [1]

(ii) For what value of k can there be an unbranched covering of $m\mathbf{T}^2 \longrightarrow k\mathbb{RP}^2$, where $m \geq 1$? [4] $y(y^4+1)$

(iii) By considering a torus symmetrically situated with respect to the origin, explain how to define an unbranched two-fold covering of $2\mathbb{RP}^2$ by $\mathbf{T}^2$. [4] y^5+y

(iv) Hence explain how to obtain the unbranched covering $m\mathbf{T}^2 \xrightarrow{2} k\mathbb{RP}^2$ of (ii) above for all appropriate pairs of values of k and m. [2]



Question 15

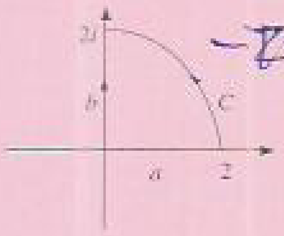
(i) Show that all five zeros of $F(z) = z^5 + z + 9$ lie inside the circle $C_2 = \{z : |z| = 2\}$. [1] $0 < V_y < \frac{\pi}{2}$

(ii) Show that $F(z)$ has no zeros inside the circle $C_1 = \{z : |z| = 1\}$. [1]

(iii) Show that $F(z)$ has exactly one real zero, and state whether it lies in the interval $]-2, -1[$ or in the interval $]1, 2[$. [1] $-\frac{\pi}{2} < V_y < 0$

(iv) Show that $F(z)$ has no zeros on the imaginary axis. [1]

(v) By considering the image under F of the contour below, find out how many zeros of $F(z)$ have both real and imaginary parts positive.



$-\pi < V(F) - \frac{5\pi}{2} < \pi$

$\frac{3\pi}{2} < V(F) < \frac{7\pi}{2}$

$\pi < VF - V_y < \frac{7\pi}{2}$

(The contour consists of a, the real axis from 0 to 2, followed by C, the quarter-circle radius 2 center the origin, from 2 to $2i$, followed by b, the imaginary axis from $2i$ to 0.) [6]

(vi) Hence determine the number of zeros of $F(z)$ in each quadrant. [1]