Question 10

A closed surface with Euler characteristic χ is to be given a regular subdivision with F faces, V vertices and E edges. Each face is to have 6 edges and k faces are to meet each vertex.

(i) Write down the condition that a closed surface subdivided into F 6-sided polygons has k faces meeting at each vertex, as a formula connecting k, V and E. [1]

(ii) Write down a formula connecting F, k, and V. [1]

(iii) Write down a formula connecting F, k, and the Euler characteristic χ. [1]

(iv) Hence find all possible regular subdivisions of this type when

(a) $\chi = 2$;

(b) $\chi = 1$;

(c) $\chi = 0$;

(d) $\chi = -1$. [8]

Your answer should give all the possible values of k, F, E, and V in each case.

$a^{-1}ba$

Question 11

Reduce each of the following edge equations (or sets of edge equations) to canonical form. Hence or otherwise classify the surface they define as a connected sum of k copies of the torus T^2, m copies of the real projective plane $\mathbb{RP}^2$, and n copies of the disc D^2, for values of k, m, and n which you should state.

(i) $ab^{-1}cadcb^{-1} = 1$. ab [3]

(ii) $aba^{-1}cdb^{-1}d^{-1}c^{-1} = 1$. [4]

(iii) $aba^{-1}c = 1$, $cded^{-1} = 1$, $efgf^{-1} = 1$. [4]

$c = ab^{-1}a^{-1}$ $\cancel{ad}$ $ab^{-1}a^{-1}ded^{-1}$

$ab^{-1}a^{-1}dfg^{-1}fd^{-1}$

Question 12

(i) Find all orientable surfaces with Euler characteristic -4, and express them in connected sum form. For each surface write down the number of boundary components it has. [2]

(ii) Sketch the orientable surface with Euler characteristic -4 and 4 boundary components. [1]

(iii) Find all non-orientable surfaces with Euler characteristic -5, and express them in connected sum form. [3]

(iv) For each of the surfaces S that you obtained in part (i), write down in connected sum form the surface $S\#\mathbb{RP}^2$. [2]

(v) Hence or otherwise determine if every non-orientable surfaces with Euler characteristic -5 is of the form $S\#\mathbb{RP}^2$, where S is an orientable surface with Euler characteristic -4. [3]

$ab^{-1}cadcb^{-1}$ $\cancel{aba}$

$aac^{-1}bdcb^{-1}$ $aba^{-1}cdb^{-1}d^{-1}c^{-1}$

$aac^{-1}bcb^{-1}d$ $aba^{-1}db^{-1}d^{-1} = 1$

$(d^{-1}a)b(b^{-1}db^{-1})$