**Question 4**

Let $T_1 = \{A, \mathcal{T}\}$ where $A = \{a, b, c\}$ and $\mathcal{T}$ is the topology

$$\{\varnothing, \{a\}, \{a, b\}, \{a, c\}, A\}$$

and let $T_2 = \{\mathbf{Z}, \mathcal{V}\}$ where $\mathbf{Z}$ is the set of all integers and $\mathcal{V}$ is the topology defined as follows: $V \in \mathcal{V}$ if $0 \notin V$ or $1 \in V$.

[You need **not** check that $\mathcal{T}$, $\mathcal{V}$ are topologies.]

(i) Prove that $f : A \longrightarrow \mathbf{Z}$ given by

$$f(a) = f(b) = 1, \quad f(c) = 0$$

is $(\mathcal{T}, \mathcal{V})$-continuous. [4]

(ii) Prove that $g : A \longrightarrow \mathbf{Z}$ given by

$$g(a) = 0, \quad g(b) = g(c) = 1$$

is not $(\mathcal{T}, \mathcal{V})$-continuous. [3]

(iii) State whether or not $h : A \longrightarrow \mathbf{Z}$ given by

$$h(a) = -1, \quad h(b) = h(c) = 0$$

is $(\mathcal{T}, \mathcal{V})$-continuous, justifying your answer. [4]

**Question 5**

Let $A$ denote the interval $(-1, 1)$ in $\mathbf{R}$ and let $\mathcal{T}$ be the topology on $A$ which consists of $\varnothing$ and all intervals of the form $(a, b)$ where $-1 \leq a < 0 < b \leq 1$.

[You need **not** verify that $\mathcal{T}$ is a topology on $A$.]

Put $T = \{A, \mathcal{T}\}$ and $H = \{\frac{1}{3}, \frac{1}{2}\}$.

(i) Prove that if $x \in A$ and $x > \frac{1}{3}$ then $x$ is a limit point of $H$ in $T$. [4]

(ii) Prove that if $x \in A$ and $x \leq \frac{1}{3}$ then $x$ is not a limit point of $H$ in $T$. [4]

(iii) Determine the closure of $H$ in $T$. [3]

**Question 6**

(i) Prove that a closed subspace of a compact space is compact. [5]

(ii) For each of the following subsets of $\mathbf{R}^2$ with the usual topology, state whether or not it is compact, justifying your answer:

$$H_1 = \{(x_1, x_2) : x_2 \geq |x_1|\};$$
$$H_2 = \{(x_1, x_2) : x_2 \geq |x_1|,\ x_1^2 + x_2^2 \leq 1\};$$
$$H_3 = \{(x_1, x_2) : x_2 \geq |x_1|,\ 0 < x_1^2 + x_2^2 \leq 1\}.$$ [6]