Question 15

(i) Show that all the roots of

$$F(z) = z^6 + z + 4$$

lie inside the annulus $A = \{z : 1 \leq |z| \leq 2\}$. [2]

(ii) Show that $F(z)$ has no real zeros. [1]

(iii) Show that $F(z)$ has no zeros on the imaginary axis. [1]

(iv) Sketch the image under F of $C_1 = \{it : 0 \leq t \leq 2\}$ as a curve in the complex plane. Deduce that $V_{C_1}(F(z))$ is approximately $-\pi$ when traced in the direction of t decreasing. [2]

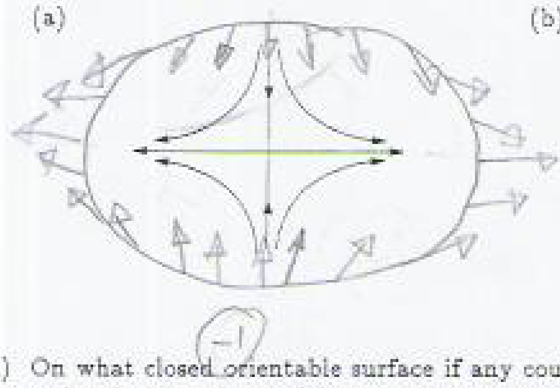
(v) Let $C_2 = \{x : 0 \leq x \leq 2\}$ and let γ denote the contour $\{z = 2e^{i\theta} : 0 \leq \theta \leq \pi/2\}$. Show that there is exactly one zero of $F(z)$ that lies in the region bounded by C_2, γ, and C_1. [4]

(vi) Deduce the number of roots of $F(z)$ lying in each quadrant. [1]

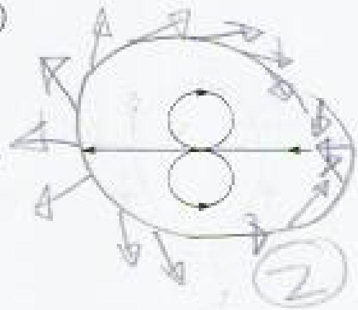
Question 16

(i) Calculate the indices of the rest points in the diagrams (a), (b) below

(a)



(b)



[4]

(ii) On what closed orientable surface if any could there be a flow with precisely 2 singularities of type (a), 1 singularity of type (b), and no others? [2]

(iii) On what closed orientable surface if any could there be a flow with precisely 4 singularities of type (a), 1 singularity of type (b), and no others? [2]

(iv) Sketch a rest point of index 1. [1]

(v) On what closed orientable surface if any could there be a flow with precisely 1 singularity of type (a), 1 singularity of type (b), and one of index 1? [2]

[END OF QUESTION PAPER]