Question 11

(i) Reduce the edge equation $e^{-1}fe^{-1} = 1$ to canonical form. Hence or otherwise classify the surface it defines as one of the following:

$$D^2, 2D^2, \mathbb{RP}^2, \mathbb{RP}^2 \# D^2, 2\mathbb{RP}^2. \qquad [2]$$

(ii) Reduce the edge equation $aba^{-1}cb^{-1}c^{-1} = 1$ to canonical form. Hence or otherwise write the surface it defines as a connected sum of the following:

$$S^2, D^2, T^2, \mathbb{RP}^2. \qquad [3]$$

(iii) Reduce the pair of simultaneous edge equations $aba^{-1}cb^{-1}d = 1$, $def^{-1}ec = 1$ to canonical form. Hence or otherwise write the surface it defines as a connected sum of the following:

$$S^2, D^2, T^2, \mathbb{RP}^2. \qquad [6]$$

Question 12

In this question all surfaces considered are orientable.
Let $A = T^2 \# D^2$ and $B = 2T^2 \# 4D^2$, so $kA = kT^2 \# kD^2$ and $kB = 2kT^2 \# 4kD^2$. We shall say that two orientable surfaces M and N are equivalent (and write $M \sim N$), if and only if there exist integers p, q, r and s such that $M \# pA \# qB = N \# rA \# sB$.

(i) Find values of p, q, r, and s which establish that $T^2 \sim 3T^2$. [4]

(ii) Either find values of p, q, r, and s which establish that $T^2 \# D^2 \sim 3T^2 \# 2D^2$, or explain why this cannot be done. [7]

Question 13

Categorize each of the following conics (which you may assume are non-degenerate) as one of the following:

(i) homeomorphic to the z-sphere;

(ii) a double cover of the z-sphere branched over two points;

(iii) a double cover of the z-sphere branched over two points and having a pinch point.

In each case locate the branch points and pinch point when they exist.

(a) $7z^2 + 8zw + w^2 + z - 2w + 1 = 0$; [4]

(b) $2z^2 + 8zw + 8w^2 - 4w + 1 = 0$; [4]

(c) $7z^2 + 4zw - 2z + 3 = 0$. [3]

Question 14

(i) Find the least value of n for a given value of k such that there is a 2-fold covering by nT^2 of kT^2. [1]

(ii) Find the least value of n for a given value of k and r such that there is an r-fold covering by nT^2 of kT^2. [1]

(iii) Show pictorially that there are unbranched 2-fold coverings

(a) by $3T^2$ of $2T^2$;

(b) by $5T^2$ of $3T^2$. [4]

(iv) Hence or otherwise describe an unbranched 4-fold covering by $5T^2$ of $2T^2$. [2]

(v) Show pictorially that there is an unbranched 3-fold covering by $7T^2$ of $3T^2$, and an unbranched 6-fold covering by $7T^2$ of $2T^2$. [3]