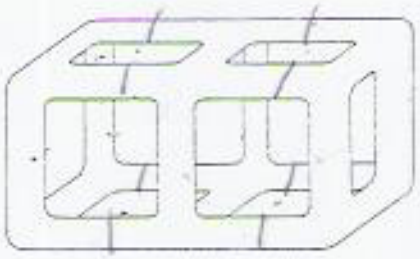
Question 9

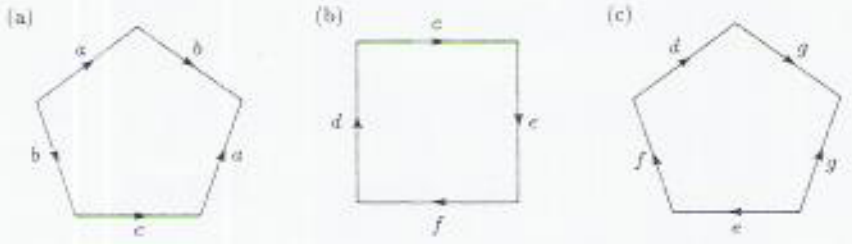
(i) The following surface (which is made of two cubes in hollow tubing joined together) is homeomorphic to an n-holed torus. Find the value of n and write down χ (the Euler characteristic) for the surface.



$3 \cdot 0 - 2 \cdot 8 = -16$

[4]

(ii) For each of the surfaces described as topological polygons below determine the corresponding number of vertices and edges the surface contains. Hence deduce whether it is a disc, a cylinder, a Möbius band, a torus, or a torus with a hole.



[3]

(iii) Consider the surface defined by the three polygons taken together. By identifying the two edges labelled c together and identifying the two edges labelled f together, redraw the polygons in (ii) above as a single topological polygon. Hence determine χ and β (the number of boundary components) for the surface they define. [3]

(iv) State whether the surface so obtained is either a torus, a torus with a hole, a Möbius band, or a Klein bottle. [1]