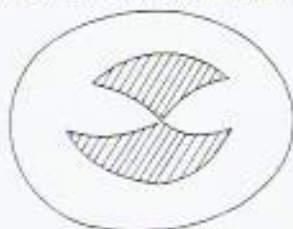


Question 12

Let S denote the surface indicated below, where the shading indicates a hole.



- (i) (a) On a copy of S , mark a subdivision, and hence find the value of its Euler characteristic, χ . [2]
- (b) Hence write down its canonical form as a connected sum of either tori and discs or else copies of $\mathbb{R}P^2$ and discs. [3]
- (ii) Write down in canonical form using the connected sum notation the result of making the connected sum of S with each of the following:
 - (a) $\mathbb{R}P^2$
 - (b) T^2
 - (c) a Möbius band. [3]
- (iii) Which non-orientable surfaces can be obtained as a connected sum of a number of copies of $\mathbb{R}P^2$ and a number of copies of S ? [3]

Question 13

For each of the following conics (which you may assume are non-degenerate) locate all branch points and the pinch point (where they exist). You need only locate those infinite points which are branch or pinch points. Hence categorize each conic as:

- (a) homeomorphic to the z -sphere because it is a single cover of it,
or
 - (b) a double cover of the z -sphere branched over two points but without a pinch point,
or
 - (c) a double cover of the z -sphere branched over two points and having a pinch point.
- (i) $z^2 + zw + 2w^2 + 4w + 2 = 0$ [4]
 - (ii) $z^2 + 6zw + 9w^2 + 4w + 2 = 0$ [4]
 - (iii) $2z^2 + zw + 4z + 1 = 0$ [3]