Question 14

(i) For each of the following coverings of T^2 by a surface S, classify the surface S and find any branch points with their deficits.

(a) $T^2 : aba^{-1}b^{-1} = 1$

$S_1 : a_1b_1a_2^{-1}b_3^{-1} = 1$

$a_2b_2a_1^{-1}b_1^{-1} = 1$

$a_3b_3a_3^{-1}b_2^{-1} = 1$ [4]

(b) $T^2 : aba^{-1}b^{-1} = 1$

$S_2 : a_1b_1a_2^{-1}b_3^{-1} = 1$

$a_2b_2a_3^{-1}b_1^{-1} = 1$

$a_3b_3a_1^{-1}b_2^{-1} = 1$ [4]

(ii) Let S' and S be two orientable surfaces without boundaries. Suppose that the surface S' is of genus g' and that it covers the surface S, which is of genus g (recall that for an orientable surfaces without boundary, the Euler characteristic χ is related to the genus p by the formula $\chi = 2 - 2p$). Show that $g' \geq g$. [3]

Question 15

(i) Show that all the zeros of $F(z) = z^4 + z^3 + z + 5$ lie inside the annulus $\{z : 1 \leq |z| \leq 2\}$. [4]

(ii) By considering $F'(z)$ show that $F(z)$ has no real zeros. [1]

(iii) Show that there are no zeros of $F(z)$ on the imaginary axis. [1]

(iv) By considering a sketch of $F(it)$, $1 \leq t \leq 2$, show that there is exactly one zero of $F(z)$ inside the contour bounding the region $\{z : 1 \leq |z| \leq 2\}$, $\operatorname{Re} z \geq 0$, $\operatorname{Im} z \geq 0$. [3]

(v) Hence deduce how many zeros of $F(z)$ lie in each quadrant. [2]