

Question 10

A closed surface with Euler characteristic χ is to be given a regular subdivision with F faces, V vertices, and E edges. Each face of the subdivision is to be a k -gon, and the subdivision is to have j edges meeting at each vertex, as this figure shows (when $j = 3$).



- (i) Write down conditions relating V to F and E to F . [2]

We shall now suppose that the subdivision has exactly one face ($F = 1$).

- (ii) Express the Euler characteristic χ as a function of j and k alone. [2]

We shall now further suppose that $\chi = -4$.

- (iii) Hence find an equation connecting j and k . [1]

- (iv) Deduce lower and upper bounds for k . [2]

- (v) Find the only possible values for k and write down the corresponding values for j , E , and V . [4]

Question 11

Reduce each of the following systems of edge equations to canonical form. Justifying each step of your answer.

- (i) $abc^{-1}dc^{-1}bd = 1$. [3]

- (ii) $aba^{-1}cdc^{-1}bd = 1$. [4]

- (iii) $ada^{-1} = 1$, $ced^{-1}c^{-1} = 1$, $beb^{-1} = 1$. [4]