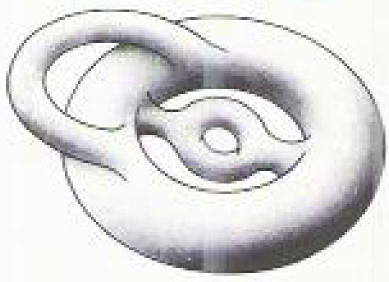
Now pulling the arm containing $AB$ over to the left gives



which is a 4-holed torus.
$\chi(S) = -6$, $\beta(S) = 0$ and $S$ is orientable.

### Question 10

(i) Each of the $F$ faces has seven edges, but each edge occurs in exactly two faces, so

$$7F = 2E.$$

At each vertex there are $k$ faces, and so $k$ edges. Thus (since each edge has two vertices)

$$\begin{aligned} kV &= 2E \\ &= 7F. \end{aligned}$$

(ii) $F - E + V = \chi = -2$.
Therefore

$$\left(\frac{2}{7} - 1 + \frac{2}{k}\right) E = -2.$$

If $k = 8$, then

$$E = -2 \Big/ \left(\frac{2}{7} - 1 + \frac{1}{4}\right) = \frac{56}{13} \text{ which is not an integer.}$$

This is impossible.

(iii) $\dfrac{2}{k} = \dfrac{5}{7} - \dfrac{2}{E}$, so

$$k = \frac{14E}{5E - 14}, \text{ and } E = \frac{14k}{5k - 14}.$$

The second formula shows that $k \geq 3$, since $E > 0$. Also, since there must be at least one face, $E \geqslant 7$. Thus

$$\frac{14k}{5k - 14} \geqslant 7,$$

$$\begin{aligned} &\text{and then } 14k \geqslant 35k - 98, \\ &\qquad \text{or } 21k \leqslant 98. \end{aligned}$$

Hence $k \leqslant 4$.
But if $k = 4$ then $E = \frac{56}{6}$ which is not an integer. If $k = 3$ then a genuine possibility exists with

$$E = 42, F = 12, V = 28.$$