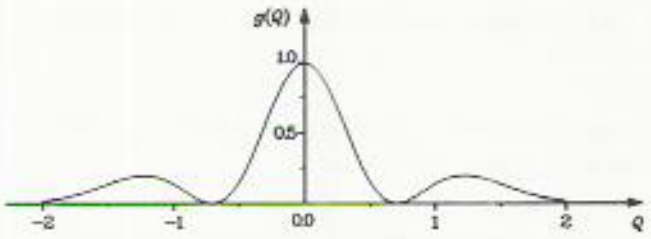

Figure 5 Sketch of the graph of the function $g(Q) = \left(1 - 2Q^2\right)^2 e^{-2Q^2}$, which is proportional to the effective potential.

From this figure we see that there are basically three possible types of motion. Suppose first that the energy E of the particle is greater than the maximum of the potential at $Q = 0$, that is,

$$\tfrac{1}{2}mv^2 = E > \frac{F^2}{4m\Omega^2} \qquad \text{or} \qquad v > \frac{F}{m\Omega\sqrt{2}}, \tag{5}$$

where v is the particle's speed sufficiently far from the origin that the effective potential may be considered zero. In this case the particle has sufficient energy to pass over the top of the barrier at $Q = 0$ and continue without being reflected; then $\dot{q} \neq 0$ so the particle passes either from left to right, $\dot{q} > 0$, or from right to left, $\dot{q} < 0$, without interruption.

If the energy is insufficient to pass over the top of the barrier at $Q = 0$, but large enough to pass over the barriers at $Q^2 = 3/2$,

$$\frac{F^2}{e^3 m\Omega^2} < E < \frac{F^2}{4m\Omega^2} \qquad \text{or} \qquad \frac{2F}{m\Omega\sqrt{2e^3}} < v < \frac{F}{m\Omega\sqrt{2}},$$

then it will be reflected from the barrier at a point given by the root of the equation

$$E = \frac{F^2}{4m\Omega^2}\left(1 - 2Q^2\right)^2 e^{-2Q^2}. \tag{6}$$

In this case the sign of $\dot{q}$ changes.

If the energy is smaller than the height of the outer barrier,

$$E < \frac{F^2}{e^3 m\Omega^2} \qquad \text{or} \qquad v < \frac{2F}{m\Omega\sqrt{2e^3}},$$

then Equation 6 will have four real roots, and the particle, if it starts a long way from the origin, will be reflected before it reaches the outer turning point.

In this case there is, in principle, another possible type of motion; that is, when the particle is in the potential well between the outer maximum and the maximum at $Q = 0$. If it were trapped here, it would execute librational motion. However, it is difficult to see how the initial conditions could be arranged to produce this type of motion; in principle it would be possible to switch the field on suddenly, just as the particle is passing over the correct spot, but such an arrangement would be very difficult in practice.

Some typical contours of this motion are shown in Figure 6. Note that the question did not ask for such a figure, but extra credit would be given for a clear sketch.