

Question 4

A new variable  $P$  is defined in terms of old conjugate variables  $(q, p)$  by

$$P = p\sqrt{q}, \quad q > 0.$$

- (i) By using an  $F_2(P, q)$  generating function, find a variable  $Q$  conjugate to  $P$ . [9]  
(ii) Hence show that conjugate variables  $(Q, P)$  can be found which convert the Hamiltonian

$$H(q, p) = \frac{1}{2}p^2 - \frac{1}{q}, \quad q > 0,$$

to the form

$$H(Q, P) = \frac{2}{Q^2}(P^2 - 2), \quad Q > 0. \quad [6]$$

- (iii) Sketch the contours of the Hamiltonian in the  $(Q, P)$ -representation, for  $Q > 0$ . [10]