(b) (i) The potential of this system, $V(q) = \frac{1}{2}q^2 - \frac{1}{3}q^3$, has a local minimum at $q = 0$ and a local maximum at $q = 1$. (This system is dealt with in some detail in *Unit 4* Subsection 4.5.5.)

So the Hamiltonian has a local minimum at $(0, 0)$ and a saddle at $(1, 0)$; since the phase curves of a Hamiltonian system are the contours of the Hamiltonian, this means that there is a stable fixed point at $(0, 0)$ and an unstable fixed point at $(1, 0)$, through which passes a separatrix. Some representative contours are shown in Figure 7.

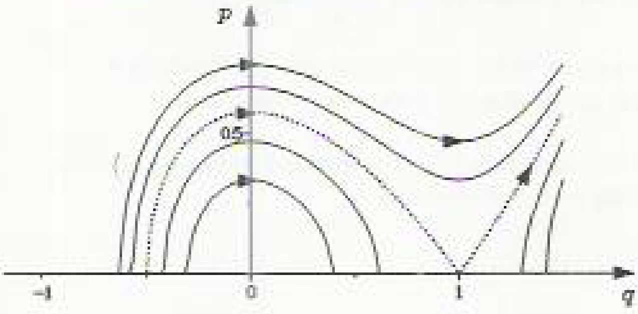

Figure 7 Some representative contours for the Hamiltonian $H(q, p) = \frac{1}{2}p^2 + \frac{1}{2}q^2 - \frac{1}{3}q^3$; only positive momenta are shown, as the contours are symmetric with respect to reflections in the q-axis.

(ii) The Melnikov function, defined in *Unit 13* Section 13.5, is most conveniently written in the form

$$M(t_0) = \int_{-\infty}^{\infty} d\tau \, \{G_1(\tau)P_2(\tau, \tau + t_0) - G_2(\tau)P_1(\tau, \tau + t_0)\},$$

where the notations $\mathbf{G}(\tau) = \mathbf{G}(\mathbf{x}_0(\tau))$ and $\mathbf{P}(\tau, t) = \mathbf{P}(\mathbf{x}_0(\tau), t)$ have been used, and where $\mathbf{x}_0 = (q_s, p_s)$ is the motion on the separatrix. On writing $\mathbf{G} = (p, -q + q^2)$ and $\mathbf{P} = (f(q, p), 0)$, we see that, since the perturbing function $\mathbf{P}(\mathbf{x}, t)$ is independent of t, the Melnikov integral must be independent of t_0. Therefore it cannot have simple zeros, and for sufficiently small ε the stable and unstable manifolds do not cross transversely; therefore there is no chaotic motion in the vicinity of the unperturbed separatrix. The reason for this is that the perturbed system is autonomous and two-dimensional.

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