The linearisation of the system about an arbitrary point is

$$\dot{\mathbf{z}} = \begin{bmatrix} 3y & 3x \\ -2x & -8y \end{bmatrix},$$

so $\operatorname{tr}\mathbf{A} = -5y$ and $\det\mathbf{A} = 6x^2 - 24y^2$.

The values of the trace and determinant of $\mathbf{A}$ at these fixed points, together with its eigenvalues, which are the solutions of the quadratic

$$\lambda^2 - \lambda \operatorname{tr}\mathbf{A} + \det\mathbf{A} = 0,$$

are shown in the following table.

$\mathbf{x}_f$	$\operatorname{tr}\mathbf{A}$	$\det\mathbf{A}$	λ
$(0,1)$	-5	-24	$3, -8$
$(0,-1)$	5	-24	$8, -3$
$(2,0)$	0	24	$\pm i\sqrt{24}$
$(-2,0)$	0	24	$\pm i\sqrt{24}$

Thus at $(0,\pm 1)$ the eigenvalues are real; as one eigenvalue is positive, this fixed point is unstable, but not strongly unstable as the other eigenvalue is negative. This classification is also valid for the non-linear system.

At $(\pm 2, 0)$ the eigenvalues are both purely imaginary, so these fixed points are centres of the linear system, and are therefore stable for the linear system; however, we can deduce nothing about the stability of the non-linear system in the vicinity of this fixed point from these results.

Question 2

(i) In any problem like this the first thing to do is to find and classify the stationary points of the potential. These are at the roots of

$$\frac{dV}{dq} = (1 - 2q^2)e^{-q^2} = 0,$$

so there are two stationary points, at $q = \pm 1/\sqrt{2}$. Now the potential is negative for $q < 0$, positive for $q > 0$, and $V(q) \to 0$ as $q \to \pm\infty$; so, as there are only two stationary points, that at $q = -1/\sqrt{2}$ must be a minimum and that at $q = 1/\sqrt{2}$ must be a maximum. The values of the potential at these extrema are

$$V(\pm 1/\sqrt{2}) = \pm \frac{1}{\sqrt{2e}}.$$

A sketch of the potential $V(q) = qe^{-q^2}$ showing these features is shown in Figure 1.

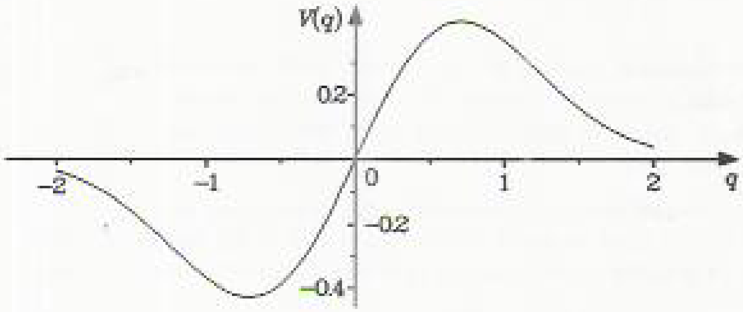

Figure 1 Graph of the potential $V(q) = qe^{-q^2}$.