Question 3

(a) A dynamical system may be represented either in (q, p)-space, in terms of a Hamiltonian function $H(q, p, t)$, or in $(q, \dot{q})$-space, in terms of a Lagrangian function $L(q, \dot{q}, t)$. Explain the connection between the two representations; include in your explanation a geometrical description of the relationship between the coordinates. [5]

(b) Find the Hamiltonian corresponding to the Lagrangian

$$L(q, \dot{q}, t; \lambda) = \tfrac{1}{2}\left(\dot{q} - e^{-\lambda q t}\right)^2$$

where λ is a parameter. Show directly that the following relation holds when H and L are partially differentiated with respect to the parameter

$$\frac{\partial H}{\partial \lambda} = -\frac{\partial L}{\partial \lambda},$$

where all other independent variables are held constant when differentiating with respect to λ. [7]

(c) If, in general, the Lagrangian $L(q, \dot{q}, t; \lambda)$ depends upon a parameter λ and $H(q, p, t; \lambda)$ is the corresponding Hamiltonian, show that

$$\frac{\partial H}{\partial \lambda} = -\frac{\partial L}{\partial \lambda}.$$ [10]

(d) State one significant difference between the flow in the $(q, \dot{q})$-state space and the (q, p)-phase space. [3]