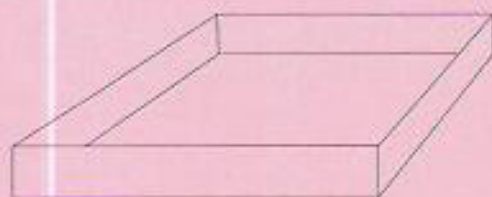


### Question 5

The outer sides of a flat, square box, with a base but no top, are to be independently coloured with the three colours red, white and blue.



Under the group of *direct* symmetries of the box, use the Counting Lemma to calculate the number of equivalence classes of the colourings. [5]

### Question 6

Let  $G$  be the group given by the direct product

$$\mathbb{Z}_3 \times \mathbb{Z}_5.$$

- (a) What are the possible orders for the subgroups of  $G$ ? [1]  
 (b) Find the number of subgroups of each possible order, justifying your answers. [4]

### Question 7

Figure 1 on the Figure Sheet is a copy of the figure below, which depicts part of a tiling.

[NOTE: For this Specimen Paper *only* the Figure Sheet is attached to the rest of the paper. It would normally be a separate item to be handed in with your answer book(s).]



- (a) On Figure 1 on the Figure Sheet, indicate the three translational tile orbits and the four translational vertex orbits by placing circled numbers in the tiles and uncircled numbers by the vertices. [3]  
 (b) Using your numbering from part (a), draw the tile-vertex diagram. [2]

### Question 8

Let  $A_1$  and  $A_2$  be two Abelian groups given by the following direct products:

$$A_1 = \mathbb{Z}_6 \times \mathbb{Z}_4 \times \mathbb{Z}_{15},$$

$$A_2 = \mathbb{Z}_{18} \times \mathbb{Z}_{20}.$$

Are  $A_1$  and  $A_2$  isomorphic? Justify your answer. [5]