

## Solutions to Part IIA

A reminder: at most two of your three questions attempted for Part II may be chosen from Part IIA.

### Question 12

- (a) There is rotational symmetry but no reflection symmetry (because of the lettering). Thus, the symmetry group  $\Gamma$  is

$$\Gamma = \{e, r, r^2, r^3, r^4, r^5\}, \quad \text{where } r = r[\pi/3].$$

For the action of  $\Gamma$  on the points of the badge,

$$P_{\Gamma}(x_1, x_2, x_3, x_6) = \frac{1}{6} (x_1^6 + 2x_6 + 2x_3^2 + x_2^3).$$

So, the number of equivalence classes using two colours is

$$\frac{1}{6} (2^6 + 2 \times 2 + 2 \times 2^2 + 2^3) = 14.$$

- (b) The pattern inventory is

$$\begin{aligned} & \frac{1}{6} [(G+S)^6 + 2(G^6+S^6) + 2(G^3+S^3)^2 + (G^2+S^2)^3] \\ &= \frac{1}{6} [G^6 + 6G^5S + 15G^4S^2 + 20G^3S^3 + 15G^2S^4 + 6GS^5 + S^6 \\ & \quad + 2G^6 \quad \quad \quad + 2S^6 \\ & \quad + 2G^6 \quad \quad \quad + 4G^3S^3 \quad \quad \quad + 2S^6 \\ & \quad + G^6 \quad \quad \quad + 3G^4S^2 \quad \quad \quad + 3G^2S^4 \quad \quad \quad + S^6] \\ &= G^6 + G^5S + 3G^4S^2 + 4G^3S^3 + 3G^2S^4 + GS^5 + S^6. \end{aligned}$$

Thus there are one each of: all gold, all silver, five gold and one silver, five silver and one gold. There are three each of: four gold and two silver, two gold and four silver. Finally there are four of three gold and three silver. [This is 14 colourings in all.]

- (c) Suppose that the third colour is black ( $B$ ). If the central hexagon is  $B$  then, as above, there are 14 colourings of the points. Similarly, if the central hexagon is  $G$ , there are 14 colourings using  $S$  and  $B$ . Similarly for  $S$  in the centre. The total is 42.

### Question 13

- (a) Highest order of rotation? 1.  
Any reflections? Yes.  
Type of reflections? Rhombic.  
So, type is:  $cm$ .
- (b) Highest order of rotation: 2.  
Any reflections? Yes.  
Reflections in two directions? No.  
So, type is:  $p2mg$ .
- (c) Highest order of rotation: 3.  
Any reflections? Yes.  
All 3-centres on reflection axes? No.  
So, type is:  $p31m$ .

$\Gamma$  to avoid confusion with  $G$  for gold. (But no marks would be deducted if you used  $G$ !)

6 About 2 each for the group, the cycle index and the final result

4

2

3

5

5

5