THE OPEN UNIVERSITY

Mathematics: Taught MSc

M827 Applicable Differential Geometry

M827 Solutions to Specimen Examination Paper

Question 1

(a) [*4 marks*]
In terms of coordinates (y^1, y^2, y^3) on the codomain manifold, say $\mathcal{N}$, the images of the coordinate vectors $\partial/\partial x^1$ and $\partial/\partial x^2$, at the point with coordinates $(0,0)$, under ϕ_* are

$$\phi_*\left(\frac{\partial}{\partial x^1}\right) = \frac{\partial}{\partial y^1} + \frac{\partial}{\partial y^2}$$
$$\phi_*\left(\frac{\partial}{\partial x^2}\right) = \frac{\partial}{\partial y^1} - \frac{\partial}{\partial y^2}$$

(tangent vectors at the point of $\mathcal{N}$ with coordinates $(0,0,0)$). The image under ϕ_* of the tangent space to $\mathcal{M}$ at the point with coordinates $(0,0)$ is thus the subspace of the tangent space to $\mathcal{N}$ at the point with coordinates $(0,0,0)$ spanned by the coordinate vectors $\partial/\partial y^1$ and $\partial/\partial y^2$.

(b) [*6 marks*]
The maps ϕ_t are defined for all t; $\phi_t(x^1, x^2)$ is smooth in all three variables.

$$\phi_0(x^1, x^2) = (x^1, x^2)$$

$$\begin{aligned}
\phi_s^1\left(\phi_t(x^1,x^2)\right) &= (x^1+t)+s = x^1+(s+t)\\
&= \phi^1_{s+t}(x^1,x^2)\\
\phi_s^2\left(\phi_t(x^1,x^2)\right) &= \left(x^2+(1-2x^1)t-t^2\right) + \left(1-2(x^1+t)\right)s - s^2\\
&= x^2 + (1-2x^1)(s+t) - t^2 - 2ts - s^2\\
&= x^2 + (1-2x^1)(s+t) - (s+t)^2\\
&= \phi^2_{s+t}(x^1,x^2).
\end{aligned}$$

Thus ϕ_t is a one-parameter group. Its infinitesimal generator, at (x^1, x^2), is the tangent vector at $t=0$ to the curve

$$t \mapsto \left(x^1+t, x^2+(1-2x^1)t-t^2\right),$$

which is

$$\partial_1 + (1-2x^1)\partial_2.$$

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