(iii) [*3 marks*]

$$\begin{aligned} R(\partial_c, \partial_d)\partial_b &= R^a{}_{bcd}\partial_a \\ &= \nabla_{\partial_c}\nabla_{\partial_d}\partial_b - \nabla_{\partial_d}\nabla_{\partial_c}\partial_b \\ &= \nabla_{\partial_c}(\Gamma^e_{bd}\partial_e) - \nabla_{\partial_d}(\Gamma^e_{bc}\partial_e) \\ &= (\partial_c\Gamma^a_{bd} - \partial_d\Gamma^a_{bc} + \Gamma^e_{bd}\Gamma^a_{ec} - \Gamma^e_{bc}\Gamma^a_{ed})\,\partial_a. \end{aligned}$$

Thus

$$R^a{}_{bcd} = \partial_c\Gamma^a_{bd} - \partial_d\Gamma^a_{bc} + \Gamma^e_{bd}\Gamma^a_{ec} - \Gamma^e_{bc}\Gamma^a_{ed}.$$

(iv) [*5 marks*]
The first Bianchi identity for a symmetric connection states that, for all vector fields U, V and W,

$$R(U,V)W + R(V,W)U + R(W,U)V = 0.$$

It may be proved as follows:

$$\begin{aligned} &R(U,V)W + R(V,W)U + R(W,U)V \\ &\quad = \nabla_U\nabla_V W - \nabla_V\nabla_U W - \nabla_{[U,V]}W \\ &\qquad + \nabla_V\nabla_W U - \nabla_W\nabla_V U - \nabla_{[V,W]}U \\ &\qquad + \nabla_W\nabla_U V - \nabla_U\nabla_W V - \nabla_{[W,U]}V \\ &\quad = \nabla_U[V,W] - \nabla_{[V,W]}U \\ &\qquad + \nabla_V[W,U] - \nabla_{[W,U]}V \\ &\qquad + \nabla_W[U,V] - \nabla_{[U,V]}W \\ &\quad = [U,[V,W]] + [V,[W,U]] + [W,[U,V]] \\ &\quad = 0 \end{aligned}$$

by the Jacobi identity. (This is an alternative to the proof given on page 275 of the textbook.)