

To apply part (i) it is necessary to check that the left-hand factor satisfies the skew-symmetry condition. From the skew-symmetry of the connection forms

$$(\mathcal{L}_X \omega_a^d) \delta_{db} + (\mathcal{L}_X \omega_b^d) \delta_{ad} = 0$$

and from the skew-symmetry of the ξ_b^a

$$(d\xi_a^d) \delta_{db} + (d\xi_b^d) \delta_{ad} = 0.$$

Finally

$$\begin{aligned} & (\omega_c^d \xi_a^c - \xi_c^d \omega_a^c) \delta_{db} + (\omega_c^d \xi_b^c - \xi_c^d \omega_b^c) \delta_{ad} \\ &= -\omega_b^d \xi_a^c \delta_{cd} + \xi_b^d \omega_a^c \delta_{cd} - \omega_a^d \xi_b^c \delta_{dc} + \xi_a^d \omega_b^c \delta_{dc} \\ &= -\omega_b^d \xi_a^c \delta_{cd} + \xi_b^d \omega_a^c \delta_{cd} - \omega_a^c \xi_b^d \delta_{cd} + \xi_a^c \omega_b^d \delta_{cd} \\ &= 0. \end{aligned}$$

Thus by part (i)

$$\mathcal{L}_X \omega_b^c + d\xi_b^c + \omega_c^a \xi_b^c - \xi_c^a \omega_b^c = 0.$$