

PART 1

- (i) You should attempt as many questions as you can in this part.
(ii) Each question in this part carries 8 marks.

Question 1

Let $w = 2/(1+i)$. Determine

- (a) $\operatorname{Arg} w$; [2]
(c) all the cube roots of w , identifying the principal cube root (express your answers in polar form); [5]
(e) the smallest positive integer n such that w^n is real. [1]

Question 2

Let

$$A = \mathbb{C} - \{x \in \mathbb{R} : x \geq 0\}, B = \mathbb{C} - \{x \in \mathbb{R} : x \leq 0\}, \text{ and } D = \{z : |z-3| < 2\}.$$

For each of the following sets:

$$A \cup B, \quad A \cap D, \quad D - \{2\}, \quad \partial B,$$

write down whether the set is

- (a) a region, [4]
(b) a closed set. [4]

Question 3

- (a) Evaluate

$$\int_{\Gamma} \operatorname{Im} z \, dz,$$

where Γ is the line segment from $-i$ to i . [3]

- (b) Determine an upper estimate for the modulus of

$$\int_C \frac{2 \sinh z}{z^5 - 1} dz,$$

where C is the circle $\{z : |z| = 2\}$. [5]

Question 4

- (a) Determine the disc of convergence of the power series

$$\sum_{n=2}^{\infty} \frac{z^n}{(n-1)!} [3]$$

- (b) Find the Taylor series about 0 (up to the term in z^3) for the function

$$f(z) = \frac{\exp(-z)}{1-z},$$

and state an open disc on which the series represents f . [5]