Question 11

Let Σ be the splitting field in $\mathbf{C}$ for the polynomial $t^4 - 4$ in $\mathbf{Q}[t]$. Let G be the Galois group $\Gamma(\Sigma : \mathbf{Q})$.

(i) Find a basis for Σ as a vector space over $\mathbf{Q}$. [6]

(ii) Find G, specifying for each element of G its effect on each of the basis elements listed in your answer to part (i). [4]

Question 12

Determine whether or not the following ruler and compass constructions are possible. In each case you should give a justification of your conclusion.

(i) Construction of a regular 1028-gon. [3]

(ii) Construction of a regular hexagon with area equal to the area of a given square. [7]

Question 13

For each of the following polynomials in $\mathbf{Q}[t]$, decide whether or not it is soluble by radicals over $\mathbf{Q}$ giving, in each case, a justification for your conclusion.

(i) $t^3 + 27t^2 - 6$ [3]

(ii) $t^5 - 4t + 2$ [7]

Question 14

Let p be prime number in $\mathbf{Z}$, let f be an irreducible polynomial of degree 6 in $\mathbf{Z}_p[t]$ and let L be a splitting field for f over $\mathbf{Z}_p$.

(i) Prove that every irreducible polynomial of degree 6 in $\mathbf{Z}_p[t]$ splits over L. [4]

(ii) Let g be a polynomial of degree 3 in $\mathbf{Z}_p[t]$. Prove that g splits over L, justifying your answer. [3]

(iii) State whether every polynomial of degree 4 in $\mathbf{Z}_p[t]$ splits over L, justifying your answer. [3]

Question 15

Let K be the subfield of $\mathbf{R}$ generated by $\mathbf{Q}$ together with the set X of all real roots of 3, i.e. the set $X = \{\sqrt[n]{3} : n \in \mathbf{R} \text{ and } n \geq 2\}$.

State, with brief reasons, whether or not the field extension $K : \mathbf{Q}$ is:

(i) algebraic; [2]

(ii) finite; [4]

(iii) simple; [3]

(iv) separable. [1]

[END OF QUESTION PAPER]