This paper is not to be removed from the Examination Halls

UNIVERSITY OF LONDON

279 005b ZA
990 005b ZA
996 D05b ZA

BSc degrees and Diplomas for Graduates in Economics, Management, Finance and the Social Sciences, the Diploma in Economics and Access Route for Students in the External Programme

Mathematics 2 (half unit)

Thursday, 11 May 2006 : 2.30pm to 4.30pm

Candidates should answer **EIGHT** of the following **TEN** questions: **SIX** from Section A (60 marks in total) and **TWO** from Section B (20 marks each). **Candidates are strongly advised to divide their time accordingly.**

Graph paper is provided. If used, it must be fastened securely inside the answer book.

Calculators may **not** be used for this paper.

PLEASE TURN OVER

SECTION A

Answer all **six** questions from this section (60 marks in total)

1. Suppose that the demand equation for a good is given by

$$p(q+4) - 50 = 0,$$

and suppose that the equilibrium quantity is 6. Calculate the consumer surplus. Suppose the supply equation takes the form $p = a + bq$ for some positive constants a and b. If the producer surplus is 12, determine a and b.

2. The function $f(x, y)$ is given by

$$f(x,y) = \frac{(x^a + y^a)}{xy} \ln\left(\frac{x}{y}\right),$$

where a is some constant. Show that f is homogeneous and verify that Euler's equation holds.

3. Suppose the demand function for a commodity is given by

$$q = \frac{10}{\sqrt{4 + 8p^3}}.$$

Find the elasticity of demand, in terms of p. Determine the values of p for which the demand is elastic.

4. Show that the following system of equations has a solution for all values of a, provided a certain relationship between c and b holds. (You should determine what this relationship is). Find all solutions, in terms of a and b, when the system has solutions.

$$\begin{aligned} x - y + 2z &= a \\ 2x - 3y + 4z &= b \\ 4x + 6z &= 15a \\ x + 3y + 8z &= c. \end{aligned}$$

PLEASE TURN OVER

5. Expand as a power series, in terms up to x^8, $f(x) = \cos(e^{x^2} - 1)$.

6. Suppose that the amount $A(t)$ of money in an investor's account at time t satisfies the differential equation

$$\frac{dA}{dt} = f(t) + rA(t),$$

where $r > 0$ is a constant and f is a function of t. Show that for any $t \geq 0$,

$$A(t) = e^{rt}A(0) + e^{rt}\int_0^t f(x)e^{-rx}dx.$$

SECTION B

Answer **two** questions from this section (20 marks each)

7.(a) Find an invertible matrix P and a diagonal matrix D such that $P^{-1}AP = D$, where A is the matrix

$$\begin{pmatrix} 5 & 3 \\ -6 & -4 \end{pmatrix}.$$

Hence, or otherwise, find the sequences x_t and y_t (for $t = 0, 1, 2, \ldots$) satisfying $x_0 = 1$ and $y_0 = 3$ and, for $t \geq 1$,

$$\begin{aligned} x_t &= 5x_{t-1} + 3y_{t-1} \\ y_t &= -6x_{t-1} - 4y_{t-1}. \end{aligned}$$

(b) Given that the function

$$f(x, y) = \frac{x^\alpha(x^2 + y^2)^\beta - (x^3 - y^3)^4(\sqrt{y})^\gamma}{x + (\sqrt{x^2 + xy})^\delta}$$

is homogeneous of degree 5, determine γ and δ, and discover what relationship must hold between α and β.

8. A manufacturer adjusts at time t the price $p = p(t)$ of his product by reference to his current inventory (stock) $I(t)$ according to the equation

$$\frac{dp}{dt} = -(I(t) - I_0),$$

where $I_0 = I(0)$ is his initial stock. The level of stock satisfies the equation

$$\frac{dI}{dt} = Q(t) - S(t),$$

where the level of production $Q(t)$ and level of sales $S(t)$ satisfy

$$Q = a - bp - c\frac{dp}{dt},$$

$$S = \alpha - \beta p - \gamma\frac{dp}{dt},$$

where $a, b, c, \alpha, \beta, \gamma$ are constants and $\beta \neq b$. Show that there is a constant solution $p(t) = p^*$ for which $Q = S$. Show that

$$\frac{d^2p}{dt^2} + (\gamma - c)\frac{dp}{dt} + (\beta - b)p = (\alpha - a).$$

Solve this equation when $\gamma - c = 3,\ \beta - b = 2,\ \alpha - a = 1$ and when the initial price is $p(0) = 1$.

Show that, in general, if $\beta > b$ and $\gamma > c$, then the price tends to p^* .

9.(a) Determine the inverse of the matrix

$$\begin{pmatrix} 0 & 2 & 1 \\ 2 & 1 & 0 \\ 1 & 0 & 2 \end{pmatrix}.$$

(b) Use the Lagrange Multiplier Method to minimize

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$$

subject to

$$x + y + z = c,$$

for $x, y, z > 0$, where c is a *positive* constant. Use your result to deduce that for $x, y, z > 0$

$$\frac{1}{3}(x + y + z) \geq \left\{\frac{1}{3}\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right)\right\}^{-1}.$$

PLEASE TURN OVER

10.(a) Find the function $y(x)$ such that $y(0) = 1$ and

$$(1 + x^3)\frac{dy}{dx} - x^2y^2 = 0.$$

(b) Suppose the supply and demand functions for a good are, respectively,

$$q^S(p) = p - 4, \quad q^D(p) = 8 - p.$$

Determine the equilibrium price and quantity.

Suppose the government imposes a percentage tax of $100r\%$, where $0 < r < 0.5$. Find the new equilibrium price and quantity in terms of r.

Find an expression, in terms of r, for the tax revenue the government obtains, and determine the value of r that maximises this tax revenue.

END OF PAPER