SECTION A

1. In a game for two players, A and B, the players simultaneously show 1, 2, 3 or 4 fingers. If the difference in the number of fingers shown is 0 or 2, then A pays B an amount equal to the total number of fingers shown. If the difference in the number of fingers shown is 1 or 3, then B pays A an amount equal to the total number of fingers shown. Calculate the outcome matrix for A.

Considering pure strategies, find the best security level for each player and show that there is no equilibrium play.

[6 marks]

2. A simple version of the game of 'nim' is played as follows. There are two players and, at the start, two piles on the table in front of them, each containing two matches. In turn the players take any (positive) number of matches from *one* of the piles. The player taking the last match loses. Sketch a game tree.

Show that the second player has a pure strategy which guarantees a win.

[6 marks]

3. Define a saddle point for a two-person zero-sum game played with pure strategies, where A's outcome matrix has entry u_{ij} corresponding to strategy A_i for A and B_j for B.

Show that, if both (A_i, B_j) and (A_k, B_l) are saddle points, then $u_{ij} = u_{kl}$.

Find all the saddle points of the matrix

$$\begin{pmatrix} -2 & 1 & 3 & 2 \\ -1 & 2 & 0 & 1 \\ -1 & -3 & -1 & 0 \end{pmatrix}.$$

[5 marks]

4. The language AB over the alphabet $\Sigma \equiv \{a, b\}$ is defined by the regular expression

$$(ba)^*(a \vee b)a$$

(i) Give all words from AB of length 4 or less.

(ii) Give two words from AB which are not in $ba^* \vee (ba)^*$.

(iii) Give a recursive definition of AB.

(iv) Draw a transition diagram for a finite automaton which accepts only the words of AB (reading from the right and terminated by a blank).

[10 marks]

5. Let Γ be the directed graph whose adjacency matrix is given below:

$$\begin{array}{c} a \\ b \\ c \\ d \\ e \\ f \end{array} \begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \end{array}$$

Thus for example there is a directed edge from c to a but none from a to c. Determine the number of directed walks of length 2 from c to b, and write them down. Calculate the in-degree and out-degree of each vertex of Γ.

Define the term 'directed Euler walk'. Find a directed Euler walk in Γ, and write down, in order, the vertices passed in this walk. [9 marks]

6. Carry out a critical path analysis for the activity given below. Find the minimum time required to complete the task. Write down the critical path as a sequence of the α_i. For each α_i not in this path, determine the float time.

Activity	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8	α_9	α_{10}	α_{11}	α_{12}	α_{13}
Time	1	3	5	4	2	8	6	3	14	4	1	4	6
Prerequisites				α_1	α_4	α_4	α_5	α_6	α_2	α_7, α_8, α_9	α_2	α_3	α_{11}, α_{12}

[10 marks]

7. (a) Using the method of sentence tableaux, decide whether the following sequent is valid:

$$\neg p, \quad (\neg q \longrightarrow \neg r) \longrightarrow p \quad \vdash \quad q \longrightarrow \neg r.$$

[3 marks]

(b) Find a disjunctive normal form for

$$q \longrightarrow (p \wedge (\neg q \vee p)).$$

Hence or otherwise find the minimal disjunctive normal form for

$$(q \longrightarrow (p \wedge (\neg q \vee p))) \longrightarrow p.$$

[6 marks]

SECTION B

8. Solve the following two-player zero-sum games, giving the optimal strategy for each player and the value of the game:

(i)

$$\begin{pmatrix} 1 & 0 \\ -1 & 2 \end{pmatrix}.$$

(ii)

$$\begin{pmatrix} 1 & -1 & 2 \\ 3 & -2 & -4 \end{pmatrix}.$$

(iii)

$$\begin{pmatrix} 1 & -1 & 2 \\ -2 & 2 & -1 \end{pmatrix}.$$

[15 marks]

9. Consider the languages X and Y over the alphabet $\Sigma \equiv \{a, b\}$ which contain the set of words of the form

$$\mathrm{X}: \quad a^n(bab)^m$$

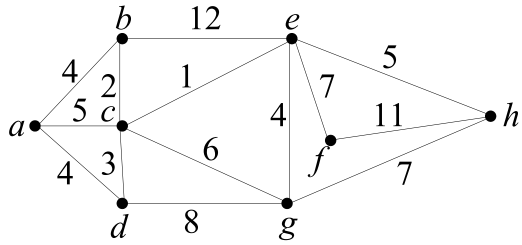
$$\mathrm{Y}: \quad a^n(bab)^n$$

where n, m are non-negative integers. (a^n denotes a string consisting of a repeated n times.)

Give productions for each of these languages and discuss the classification of the grammar in each case.

Describe two machines: one which accepts only words of the language X and the other which accepts only words of the language Y. You may assume in each case that the machine starts at the rightmost element of the word and that the word is terminated by a blank at the left. [15 marks]

10. (i) Define the term 'minimal spanning tree' of a connected weighted graph. Using the greedy algorithm, find *all* the minimal spanning trees of the graph displayed, explaining briefly why your list is complete. Verify that all your trees have the same total weight. [8 marks]



(ii) Define the term 'vertex colouring' of a graph. Taking the vertices of the displayed graph in alphabetical order, and using the greedy algorithm, how many colours are needed for a vertex colouring? (For this, the weighting is ignored.) Find, if possible, a colouring with fewer colours, or else show that no such colouring exists. [7 marks]

11. (a) Use the method of sentence tableaux to say what you can about the following argument:

$$\forall x(F(x) \longrightarrow \exists y(G(x,y) \longrightarrow H(y)))$$
$$\exists x(F(x) \wedge \forall y(B(y) \longrightarrow G(x,y)))$$
$$\vdash \exists y(B(y) \longrightarrow H(y)).$$

[8 marks]

(b) Write the following argument in symbolic form and decide whether or not it is valid.

When the wind blows, then some cradle will rock;
If some cradle rocks, then some baby will sleep;
Therefore if no baby is sleeping, the wind is not blowing.

[7 marks]