All questions carry equal weight. If a question has several parts, each part will carry equal weight. The best **ten** answers will be taken into account.

1. (a) Calculate and reduce to lowest terms:

(i) $\frac{1}{3}+\frac{1}{12}$; (ii) $\frac{1}{2}-\frac{1}{6}$;

(iii) $\frac{2}{7}\times\frac{14}{8}$; (iv) $(-\frac{5}{7})\div(-\frac{10}{21})$.

(b) Simplify

(i) $\frac{3^{27}}{3^{24}}$; (ii) $\frac{9^5}{27^3}$.

2. Simplify

(i) $\frac{2}{x+1}-\frac{2}{x-1}$; (ii) $x+2+\frac{5}{x}$;

(iii) $\frac{x}{(x+1)^2}-\frac{2}{(x+1)}$; (iv) $\frac{x}{(x-1)}+\frac{1}{(1-x)}$.

3. (a) Simplify the following:

(i) $4p-2rs+4p+2rs$; (ii) $2p^2+pr-p^2-2pr$;

(iii) $p+3rs+rs-2p$; (iv) $3(x+y)-2(x-y)$.

(b) Work out $\frac{x}{y}+\frac{y}{x}$; $\frac{x}{y}-\frac{y}{x}$; $\frac{x}{y}\times\frac{y}{x}$ and $\frac{x}{y}\div\frac{y}{x}$.

4. (a) Factorise the following quadratics and hence find their roots:

(i) x^2-x-6; (ii) $3x^2+10x+3$.

(b) Use the formula to solve the following quadratics:

(i) $x^2-4x+1=0$; (ii) $3x^2-3x-5=0$.

5. (a) Solve the inequalities:

(*i*) $-4x+5>-3$ and (*ii*) $x+1>2x-1$.

(b) Use the binomial theorem to find:

(*i*) $(x-2y)^4$ and (*ii*) $(2x+3y)^3$.

6. (a) In each of the following find the value of x which satisfies the equation:

(*i*) $\log_x 16 = 4$ (*ii*) $\log_2 x = 5$.

(b) Let $y = 2\log_a 24 - 3\log_a 4$. Express y as a single logarithm and find y when $a = 3$.

7. (a) In a class of 30 students, 16 study Physics and 15 study Chemistry. If 3 students study both Physics and Chemistry, how many students study neither ?

(b) Decide which (if any) of the following sets are equal:

$A = \{n \text{ in } \mathbf{Z} : -1 < n < 4 \text{ and } n^2 < 2n+1\}$; $B = \{n \text{ in } \mathbf{R} : n^2 = n\}$;

$C = \{n \text{ in } \mathbf{R} : -1 < n < 4 \text{ and } n^2 < 2n+1\}$;

$D = \{n \text{ in } \mathbf{Z} : n > -1 \text{ and } n^2 < 2\}$.

8. (a) Prove, by mathematical induction, that for all natural numbers n

$$2+4+6+\cdots+(2n) = n^2+n.$$

(b) A sequence of real numbers is determined by

$$a_1 = 2; \quad a_2 = 4; \quad a_{n+1} = 4a_n - 4a_{n-1} (\text{for } n > 1\ ).$$

Use induction to show that $a_n = 2^n$.

9. (a) A class of 12 students have a swimming race. How many possible results (first, second, third) are there?

(b) In how many ways can a committee of 5 be formed from 12 people?

(c) How many distinct arrangements of the four letters A,B, C and C are there?

(d) In how many ways can 6 identical coins be distributed between 4 children?

10. A die is rolled 6 times and the sequence of faces noted. How many sequences are possible? How many have "6" occuring exactly three times? In how many sequences does "6" occur an even number of times?

11. Write down the truth tables for the expressions

$$(i) \quad (p \to q) \wedge (q \to p); \quad (ii) \quad (\neg p) \wedge (p \wedge q)$$

and decide whether either is a contradiction.

12. In the set of positive integers, decide which of the following are true and which are false

(i) $\forall x(\text{ if } x < 5 \text{ then } x^2 > 4)$; (ii) $\exists x(\text{ if } x < 5 \text{ then } x^2 > 4)$;
(iii) $\exists y(\text{ if } y < 2 \text{ then } (\forall x(\text{ if } x < 3 \text{ then } y < x)))$;
(iv) $\forall y(\text{ if } y < 2 \text{ then } (\exists x(\text{ if } x < 3 \text{ then } y < x)))$.

13. Let Γ be the directed graph with 5 vertices a, b, c, d and e and adjacency matrix (with rows and columns labelled by a, b, c, d, e in that order)

$$\begin{pmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{pmatrix}$$

Draw the graph of Γ. Write down the in-degree and the out-degree at each vertex. Is this a simple graph? Is there a walk from vertex a to vertex d? Is Γ a connected graph?

14. Let Γ be a graph with v vertices and e edges. Write down a formula relating e to the degrees of the graph at each vertex of Γ. Now suppose that Γ is a tree, write down a relationship between e and v. Suppose that Γ is a tree with 2 vertices of degree 1, 2 vertices of degree 2 and k vertices of degree 4. Determine k and draw Γ.

15. Given the 2×2 matrix

$$A = \begin{pmatrix} -1 & 17 \\ 0 & 1 \end{pmatrix}$$

calculate A^2, $\det A$ and A^{-1}. What is A^4?