All questions carry equal weight. If a question has several parts, each part will carry equal weight. The best **eight** answers will be taken into account.

**1.** (a) Simplify the following:

(*i*) $5p - 3rs + 5p + 3rs$; (*ii*) $p^2 + 2pr - 2p^2 - 3pr$;

(*iii*) $2p + 6rs + 2rs - 4p$; (*iv*) $2(x + y) - (x - y)$.

(b) Simplify

(*i*) $\frac{2^{27}}{2^{23}}$; (*ii*) $\frac{4^7}{8^4}$.

**2.** Simplify

(*i*) $\frac{x}{x+1} - \frac{x}{x-1}$; (*ii*) $\frac{7}{x^2} + \frac{5}{x}$;

(*iii*) $\frac{x}{(x-1)(x+1)} - \frac{2}{(x-1)(x+2)}$ (*iv*) $\frac{x}{(x-1)} + \frac{1}{(1-x)}$.

**3.** (a) Work out $\frac{x}{y} + \frac{-y}{x}$; $\frac{x}{y} - \frac{-y}{x}$; $\frac{x}{y} \times \frac{-y}{x}$ and $\frac{x}{y} \div \frac{-y}{x}$.

(b) Solve the inequalities:

(*i*) $-2x + 3 > -1$ and (*ii*) $x - 1 > 2x + 1$.

**4.** (a) Factorise the following quadratics and hence find their solutions:

(*i*) $x^2 - x - 12$; (*ii*) $2x^2 + 5x + 2$.

(b) Use the formula to solve the following quadratics:

(*i*) $x^2 + 4x + 1 = 0$; (*ii*) $3x^2 + 2x - 5 = 0$.

**5.** (a) Let $y = 2\ \log_a 12 - log_a 9$. Express $y$ as a single logarithm and find $y$ when $a = 2$.

(b) A coin is tossed 4 times. Using "T" for tails and "H" for heads, write out all possible outcomes. How many of these outcomes have exactly two heads and two tails?

**6.** (a) In a class of 32 students, 16 study French and 15 study German. If 3 students study both French and German, how many students study neither language?

(b) Decide which of the following sets are equal:
$A = \{n \text{ in } \mathbf{Z} : 0 < n < 8 \text{ and } n^2 < 2n+1\}$; $B = \{n \text{ in } \mathbf{Z} : n^3 = n\}$;
$C = \{0, 1, 2\}$; $D = \{n \text{ in } \mathbf{Z} : n^2 < 2\}$.

**7.** (a) Use the binomial theorem to find:
$(i)$ $(2x+y)^4$ and $(ii)$ $(x-2y)^3$.

(b) Prove, by mathematical induction that for all natural numbers $n$

$$1 + 3 + 5 + \cdots (2n-1) = n^2.$$

**8.** (a) A race has 8 competitors. How many possible results (first, second, third) are there?

(b) In how many ways can a committee of 4 be formed from 10 people?

(c) How many distinct arrangements of the four letters H,A, L and L are there?

(d) In how many ways can 8 identical coins be distributed between 6 children?

**9.** Write down the truth tables for the expressions

$$(i)\quad (p \to q) \to (q \to p); \quad (ii) \quad \neg(p \to q) \to (\neg p \vee \neg q)$$

and decide whether either is a tautology.

**10.** In the set of all integers, let $p(x)$ be the predicate "$x > 1$" and $q(x)$ be the predicate "$x < 6$". Decide which of the following are true and which are false

$(i)\quad \forall x(p(x) \vee q(x));\qquad (ii)\quad \forall x(p(x)) \vee \forall x(q(x));$

$(iii)\quad \exists x(p(x) \wedge q(x));\qquad (iv)\quad \exists x(p(x)) \wedge \exists x(\neg q(x)).$

**11.** Draw a graph to satisfy each of the following specifications or indicate why it is impossible to do so

A simple graph with 4 vertices and 3 edges;

A graph with 4 vertices and 7 edges;

A simple graph with 4 vertices and 7 edges;

A connected graph with 4 vertices and 2 edges.

**12.** Let $\Gamma$ be a graph with $v$ vertices and $e$ edges. Write down a formula relating $e$ to the degrees of the graph at each vertex of $\Gamma$. Now suppose that $\Gamma$ is a tree, write down a relationship between $e$ and $v$. Suppose that $\Gamma$ is a tree with 4 vertices of degree 1, 1 vertex of degree 2 and $k$ vertices of degree 4. Determine $k$ and draw $\Gamma$

**13.** Given the $2 \times 2$ matrix

$$A = \begin{pmatrix} -1 & -1 \\ 0 & 1 \end{pmatrix}$$

calculate $A^2$, $\det A$ and $A^{-1}$. What is $A^3$?