Candidates should answer the WHOLE of Section A and THREE questions from Section B. Section A carries 55% of the available marks.

## SECTION A

**1.** The Stefan-Boltzmann constant is given by

$$\sigma = \frac{2\pi^5 k^4}{15c^2h^3} \approx 5.67 \times 10^{-8} \text{ W m}^{-2}\text{K}^{-4}.$$

Find $\sigma$ to *five* significant figures given that $k = 1.38066 \times 10^{-23}$ J K$^{-1}$, $c = 2.99792 \times 10^8$ m s$^{-1}$ and $h = 6.62608 \times 10^{-34}$ J s.

[5 marks]

**2.** Simplify

$$\frac{(a^3b)^2(b^2c^3)^4}{(c^3a^2)^3}.$$

[4 marks]

**3.** Find 2 values of $\theta$ between 0 and $2\pi$ which satisfy $\sin\theta = 0.76543$.
Give your answers in radians to 3 significant figures. [4 marks]

**4.** Write down the common difference for the arithmetic series

$$\frac{41}{3} + \frac{39}{3} + \frac{37}{3} + \cdots + \frac{1}{3}.$$

How many terms are there in this series?
Find the sum of the series.

[5 marks]

**5.** Evaluate the sum of the infinite geometric series

$$\frac{1}{5} + \frac{1}{5^2} + \frac{1}{5^3} + \cdots$$

[4 marks]

**6.** Solve the linear equations

$$5x + 4y = 23, \qquad 4x - 5y = 2.$$

[4 marks]

**7.** By completing the square, find the two roots of the equation

$$x^2 - 8x - 384 = 0.$$

[5 marks]

**8.** Find the first and second derivatives with respect to $x$ of $\sin(x^2 + 1)$.

[5 marks]

**9.** By using the substitution $u = x^2 + 9$, evaluate the integral

$$\int_0^4 x(x^2 + 9)^{1/2} \mathrm{d}x.$$

[5 marks]

**10.** Find the integral with respect to $x$ of

$$\frac{6}{2x - 3}.$$

[3 marks]

**11.** Find all the second order partial derivatives of $f(x, y) = (x^2 - y^2)^3$.

[5 marks]

**12.** Given the complex numbers $z_1 = 4 - 3j$ and $z_2 = 4 + 4j$, find $(i)$ $|z_1|$, $(ii)$ $z_1 + 2z_2$, $(iii)$ $1/z_1$ and $(iv)$ $z_1 z_2$. [6 marks]

## SECTION B

**13 (a).** The series $S(x)$ is given by

$$S(x) = \sum_{r=0}^{5} \frac{5!}{r!(5-r)!} x^r.$$

Expand this series and simplify your result.

[7 marks]

**(b).** Given

$$\cosh x = \frac{1}{2}\left(e^x + e^{-x}\right), \quad \sinh x = \frac{1}{2}\left(e^x - e^{-x}\right),$$

show that

$$\cosh^2 x - \sinh^2 x = 1 \quad \text{and} \quad 2\sinh x \cosh x = \sinh 2x.$$

[8 marks]

**14 (a).** Evaluate the integral

$$\int_0^{\frac{\pi}{2}} \theta \sin 2\theta \; d\theta.$$

[9 marks]

**(b).** Given that

$$\frac{(2x^2 + 10x + 14)}{(x+1)(x+2)(x+3)} \equiv \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{x+3},$$

find $A$, $B$ and $C$.

[6 marks]

**15.** Find and classify the stationary points of

$$y = f(x) = x^3 - 6x^2 + 9x - 2,$$

Find the point of inflection and show that it lies on the $x-$axis.
Hence write down one factor of $f(x)$.
Find the quadratic factor of $f(x)$.
Hence, find the 2 non-integer roots of $f(x)$, giving your answer to one decimal place.
Sketch $f(x)$. [15 marks]

**16 (a).** Use de Moivre's theorem to show that

$$\sin 3\theta = 3\cos^2\theta\sin\theta - \sin^3\theta.$$

Obtain a similar expression for $\cos 3\theta$. Deduce that

$$\cot 3\theta = \frac{1-3t^2}{t(3-t^2)}.$$

where $t = \tan\theta$. [7 marks]

**(b).** Simplify $e^{j\pi}$. Find the five roots of $z^5 = -1$.

Hence, or otherwise, show that

$$z^5 + 1 = (z+1)(z^2 - 2\cos(\pi/5)z + 1)(z^2 - 2\cos(3\pi/5)z + 1).$$

[8 marks]

**17.** Sketch the triangle, $\mathcal{R}$, with sides given by $x = 1$, $y = 0$ and $y = x$.
A non-uniform lamina with shape $\mathcal{R}$ has a mass density of $\rho = xy$ kg m$^{-2}$.
Show that its mass is 0.125 kg.
Find the coordinates of its centre of mass. [15 marks]