Candidates should answer the WHOLE of Section A and THREE questions from Section B. Section A carries 55% of the available marks.

SECTION A

1. The period, T, for a simple pendulum of length L is given by

$$T = 2\pi\sqrt{\frac{L}{g}},$$

where $g = 9.81\ \mathrm{ms}^{-2}$ is the acceleration due to gravity.

Find the period, to two decimal places, of

(i) a simple pendulum of length 0.5 m and

(ii) a simple pendulum of length 2.0 m.

[4 marks]

2. Simplify

$$(6x^3y^{\frac{5}{2}}z^{\frac{1}{4}})^2 \div (16x^6y^4z^3)^{\frac{1}{2}}.$$

[4 marks]

3. Find two values of θ between 0 and 2π which satisfy $\tan\theta = 30$.

Give your answers in radians to 3 significant figures. [3 marks]

4. Find the sum of the first 100 odd integers. [4 marks]

5. For the geometric series

$$1 + 1.1 + 1.21 + 1.331 + \cdots$$

find the 7th term and the sum of the first 10 terms.

Give your answers to four decimal places. [4 marks]

6. Solve the linear equations

$$4x + 2y = 5, \qquad 3x + y = 9.$$

Check your answer. [5 marks]

7. Solve the following equation for x:

$$\sqrt{14 - x} = x - 2.$$

[5 marks]

8. Sketch the graph of $y = (x-1)^2 - 1$. [4 marks]

9. Determine the first and second derivatives with respect to x of

$$x\cos(2x+1)$$

.

[4 marks]

10. Evaluate the integral

$$\int_0^1 (1-x)^2 \, \mathrm{d}x.$$

[4 marks]

11. Determine the integral

$$\int \frac{6}{3x+2} \, \mathrm{d}x.$$

[3 marks]

12. Determine all the second order partial derivatives of

$$f(x,y) = x^2 + 3xy - y^2$$

.

[5 marks]

13. Given the complex numbers $z_1 = 1 + 2j$ and $z_2 = 2 - j$,
find (i) $|z_1 - z_2|$, (ii) $1/z_1$ and (iii) $z_1 z_2$. [6 marks]

SECTION B

14. (a) Using Pascal's triangle, or otherwise, expand

$$(2a+b)^4 \ .$$

Check your answer for the case $a = 1$, $b = -1$. [7 marks]

(b) Find the first and second derivatives, with respect to t of

$$I(t) = \mathrm{e}^{-t}(2\cos 2t + \sin 2t) \ .$$

Verify that $I(t)$ satisfies

$$\frac{\mathrm{d}^2 I}{\mathrm{d}t^2} + 2\frac{\mathrm{d}I}{\mathrm{d}t} + 5I = 0 \ .$$

[8 marks]

15. (a) Using the substitution $u = x^2 + 1$, or otherwise, find the integral

$$\int (x^2+1)^{\frac{3}{2}} \ x \ \mathrm{d}x.$$

[7 marks]

(b) Using integration by parts show that

$$\int_0^\infty t\mathrm{e}^{-t} \ \mathrm{d}t = 1.$$

[8 marks]

16. Express

$$y = f(x) = (x+1)(1-x)(x-3)$$

as a cubic polynomial. [2 marks]

Hence, or otherwise, find and classify the stationary points of y and find the point of inflection. [9 marks]

Sketch $f(x)$. [4 marks]

17. (a) Given that $z = \cos\theta + j\sin\theta$, show that

$$2\cos\theta = z + z^{-1}.$$

Hence show that

$$\cos^3\theta = \frac{1}{4}(\cos 3\theta + 3\cos\theta)$$

and

$$\int_0^{\frac{\pi}{2}} \cos^3\theta \; \mathrm{d}\theta = \frac{2}{3}.$$

[9 marks]

(b) Write down the value of $\mathrm{e}^{2k\pi j}$, where k is an integer.

Find the six roots of $z^6 = 1$. Give your roots in cartesian form. [6 marks]

18. Sketch the triangular region $\mathcal{R}(y < x < 2 - y,\ 0 \le y \le 1)$.

Show that

$$M = k \int\int_{\mathcal{R}} 2xy \; \mathrm{d}x\mathrm{d}y = \frac{2}{3}k \ ,$$

where k is a constant.

[8 marks]

Evaluate the integral

$$M\bar{y} = k \int\int_{\mathcal{R}} 2xy^2 \; \mathrm{d}x\mathrm{d}y \ .$$

Given that $M\bar{x} = \frac{11}{15}k$, find $\bar{x}$ and $\bar{y}$ and suggest a possible physical interpretation for them. [7 marks]