SECTION A

1. Given $\mu_0 = 4\pi \times 10^{-7}$ N A^{-2} and $\epsilon_0 = 8.854188 \times 10^{-12}$ F m^{-1}, express $(\mu_0\epsilon_0)^{-1/2}$ in standard form, accurate to 6 significant figures.

[4 marks]

2. Simplify

(i)
$$\frac{(a^2b^3)^2}{(b^2c^3)^3(ca^{-2})^4},$$

(ii)
$$\frac{1}{2}\left[\frac{1}{s-1} + \frac{1}{s+1}\right].$$

[5 marks]

3. Evaluate

$$\sin^{-1}(0.84147) \quad \text{(in radians to 4 decimal places).}$$

[4 marks]

4. Using $\cos(A + B) = \cos A \cos B - \sin A \sin B$, find $\cos\left(\frac{5}{6}\pi\right)$, expressing your answer in surd form.

[4 marks]

5. An algebraic series (progression) has a first term of 2 and a common difference of $\frac{1}{2}$. Find the sum of the first 9 terms.

[4 marks]

6. A geometric series (progression) has a first term of 2 and a common ratio of $\frac{1}{2}$.

Find the sum of the first 9 terms, giving your result as a rational number.

[5 marks]

7. Solve the linear equations

$$2x + 3y = 9, \qquad 3x - 7y = 2.$$

[4 marks]

8. Use the standard formula to find the two roots of the equation

$$x^2 - 7x + 5 = 0.$$

Give your answer to 4 decimal points. [5 marks]

9. Find the second derivative with respect to t of $t\mathrm{e}^{-2t}$. [4 marks]

10. Evaluate

$$\int_2^4 (x-2)^2 \mathrm{d}x.$$

[4 marks]

11. Find the inverse function, $\mathrm{f}^{-1}(x)$, of

$$\mathrm{f}(x) = \frac{x-4}{x-2}.$$

Sketch $\mathrm{f}(x)$ and $\mathrm{f}^{-1}(x)$ on separate graphs. [5 marks]

12. Show that $u(x, y) = \mathrm{e}^{nx} \sin(ny)$ satisfies Laplace's equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

for any value of the constant n. [4 marks]

13. Find, by completing the square, the two complex roots of

$$z^2 - 4jz - 13 = 0.$$

Show these points on the Argand diagram. [5 marks]

SECTION B

14 (a). The series $S(x)$ is given by

$$S(x) = \sum_{r=0}^{4} \frac{4!}{r!(4-r)!} x^r.$$

Write down the series in full. Simplify it.
Hence, or otherwise, evaluate $S(\frac{1}{2})$, expressing your answer as a rational number. [7 marks]

(b). Express $\cosh x$ and $\sinh x$ in terms of the exponential function.
Show that

$$\frac{\mathrm{d}}{\mathrm{d}x}(\sinh x) = \cosh x.$$

Show that $y(t) = A\sinh(2t) + B\cosh(2t)$ satisfies the differential equation

$$\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} - 4y = 0.$$

Find the values of A and B, given that the initial conditions are $y(0) = 0$ and $y'(0) = 4$. [8 marks]

15 (a). Show that

$$\int_0^{\frac{\pi}{2}} \theta^2 \sin 3\theta \;\mathrm{d}\theta = -\frac{1}{27}(3\pi + 2).$$

[8 marks]

(b). Given that

$$\frac{(x-4)}{(x-2)(x-3)} \equiv \frac{A}{x-2} + \frac{B}{x-3},$$

find A and B.
Using this result, or otherwise, evaluate the integral

$$\int_4^6 \frac{(x-4)}{(x^2 - 5x + 6)} \mathrm{d}x.$$

Express your answer as a single logarithm. [7 marks]

16. Show that $x = 5$ is a root of the cubic

$$y = \mathrm{f}(x) = x^3 - 12x^2 + 45x - 50.$$

Hence, or otherwise, find the factors of $\mathrm{f}(x)$.
Find and classify the stationary points of $\mathrm{f}(x)$.
Find the point of inflection.
Sketch $\mathrm{f}(x)$. [15 marks]

17 (a). Express (i) $\mathrm{e}^{jn\theta}$ and (ii) $\mathrm{e}^{-jn\theta}$ in terms of $\cos n\theta$ and $\sin n\theta$.
Expand

$$(\mathrm{e}^{jn\theta} + \mathrm{e}^{-jn\theta})^5.$$

Hence, or otherwise, show that

$$2^4 \cos^5\theta = \cos 5\theta + 5\cos 3\theta + 10\cos\theta.$$

[8 marks]

(b). Find the six roots of $z^6 - 1 = 0$. Express your answer in cartesian form.
Show these numbers as points on the Argand diagram. [7 marks]

18 (a). Given that $u = x - ct$ and $v = x + ct$, show that

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u} + \frac{\partial z}{\partial v},$$

where c is a constant and find a similar expression for $\frac{\partial z}{\partial t}$.
Show that

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2} + 2\frac{\partial^2 z}{\partial u \partial v}$$

and find a similar expression for $\frac{\partial^2 z}{\partial t^2}$.
Deduce that the wave equation

$$\frac{\partial^2 z}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 z}{\partial t^2}$$

may be written

$$\frac{\partial^2 z}{\partial u \partial v} = 0.$$

[8 marks]

(b). Evaluate the double integral

$$\int\int_{\mathcal{R}} (x - y)^2 \mathrm{d}x\mathrm{d}y$$

over the region $\mathcal{R}$ $(x \le y \le 1,\ 0 \le x \le 1)$.
Sketch the region $\mathcal{R}$. [7 marks]