Answer all of section A and THREE questions from section B. The marks shown against questions, or parts of questions, indicate their relative weights. Section A carries 55% of the total marks.

SECTION A

1. Write down the natural domain and the range of each of the following functions and sketch their graphs:

   (i) $y = |x+2|$ (ii) $y = 1/(2x-1)^2$. (4 marks)

2. If

$$f(x) = \frac{1+7x}{1+x},$$

find the corresponding inverse function $f^{-1}(x)$. Verify that

$$f(f^{-1}(x)) = x.$$ (5 marks)

3. State, giving reasons, whether or not the following functions are odd, even or neither:

   (i) $f(x) = \dfrac{x^3+2x}{x^5+3x^3+x}$ (ii) $g(x) = \dfrac{x^3+\tan x}{\cos x}$. (4 marks)

4. Find the following limits

   (i) $\displaystyle\lim_{x\to 1}\frac{x^3-8x^2+7}{x+1}$ (ii) $\displaystyle\lim_{x\to -2}\frac{x^2-x-6}{x^2+5x+6}$ (iii) $\displaystyle\lim_{x\to 1}\frac{\ln x}{2x^4-2}$. (6 marks)

5. Write down the quotient rule for differentiation. Prove this rule by differentiation from first principles. (4 marks)

6. Differentiate the following identifying any rules of differentiation that you use

   (i) $y = t^4\sec(2t)$ (ii) $y = \dfrac{1+2x}{1+x^2}$ (iii) $y = \cos(\exp(-2x))$. (6 marks)

7. Use implicit differentiation to find expressions for dy/dx and d^2y/dx^2 in terms of x and y given that

$$xy + y^2 = -3x.$$ (6 marks)

8. Write down the definition of $\cosh x$ in terms of e^x and e^{-x}. Hence show that

$$\cosh^{-1}(x) = \ln\left(x+\sqrt{x^2-1}\right) \quad x \geq 1.$$ (5 marks)

9. Find the following indefinite integrals

(i) $\int \tan^8 x \sec^4 x \, dx$ (ii) $\int t^2 e^t \, dt$. (6 marks)

10. Find the arc length of the curve $y = x^{3/2}$ from $x = 0$ to $x = 8/9$. (5 marks)

11. Use the integral test to determine whether or not the following infinite series converges

$$\sum_1^\infty \frac{1}{n^7}.$$ (4 marks)

SECTION B

12. (i) Given that $x = a$ satisfies the equation $f(x) = x$, verify that $x = a$ also satisfies the equation $f(f(x)) = x$.

(ii) The function *f* is defined by

$$f(x) = x(3x - 2).$$

Find solutions of the equation

$$f(x) = x.$$

Verify that $f(f(x)) = 27x^4 - 36x^3 + 6x^2 + 4x$ and use the result of (i) to find all the solutions of the equation $f(f(x)) = x$.

(iii) Apply the Newton-Raphson method with initial value $x_0 = 0.5$ directly to $f(f(x)) - x = 0$. Hence find a solution of this equation correct to 3 decimal places. Compare your answer to that obtained in (ii) (15 marks)

13. Let

$$f(x) = \frac{x^2 + x - 5}{x - 5}.$$

Find constants A, B, and C such that

$$f(x) = Ax + B + \frac{C}{x-5}.$$

Find intervals of x on which the function is (i) increasing, (ii) decreasing, (iii) concave up and (iv) concave down. Locate any zeros, asymptotes, extrema and inflection points. Classify the extrema. Sketch the graph. (15 marks)

14. A marine installation 0.2 miles from a (straight) coast is to be connected by two straight lengths of pipe to an oil refinery 0.6 miles distant along the coast. One length of pipe (at sea) costs 0.5 per mile, in appropriate monetary units, and the other length (on land) costs 0.2 per mile in the same units. Find where the pipe should be brought ashore so as to minimise costs. Find the minimum cost and show that it is indeed a minimum. (15 marks)

15. Obtain approximate values for the integral

$$\int_0^1 \frac{3}{1+x^2} dx$$

using (i) the trapezoidal rule and (ii) Simpson's rule with the interval $[0,1]$ subdivided into ten equal parts in each case. Give your answers correct to five decimal places. Evaluate the integral exactly (you may wish to use the substitution $x = \tan\vartheta$). Compare your answers and comment very briefly on your findings. (15 marks)

16. (i) Find the derivative of the function $\tan^{-1} x$ (the inverse of the function $\tan x$).

(ii) Find the following indefinite integral

$$\int \frac{x dx}{1+x^2}.$$

(iii) Use integration by parts and results from (i) and (ii) to evaluate the following definite integral

$$\int_0^1 \tan^{-1} x \; dx.$$ (15 marks)