PAPER CODE NO.
MA06

THE UNIVERSITY
of LIVERPOOL

SUMMER 1997 EXAMINATIONS

Degree of Bachelor of Science : Year 0
Degree of Bachelor of Science : Year 1
Degree of Bachelor of Engineering : Year 0

VECTORS AND KINEMATICS

TIME ALLOWED : Three Hours

INSTRUCTIONS TO CANDIDATES

Answer ALL questions in Section A and THREE questions from Section B.
The total of the marks available on Section A is 55.

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SECTION A

1. Let $\overrightarrow{OA} = (3, -1-2)$ and $\overrightarrow{OB} = (2, -2, 3)$. Find

   (a) $\overrightarrow{AB}$;

   (b) the length of the line AB;

   (c) $\overrightarrow{OM}$, where M is the point on AB such that $3\overrightarrow{AM} = \overrightarrow{MB}$.

   [11 marks]

2. Suppose that A, B, C, D and E are five points in space and that $\overrightarrow{AB} = \mathbf{a}$, $\overrightarrow{BC} = \mathbf{b}$, $\overrightarrow{CD} = \mathbf{c}$ and $\overrightarrow{DE} = \mathbf{d}$. Find expressions for $\overrightarrow{EA}$ and $\overrightarrow{CE}$ in terms of $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ and $\mathbf{d}$.

   [5 marks]

3. Let A and B be the points $(3, 5, 2)$ and $(-1, 3, 1)$. Write down the vector equation of the line AB.

   [7 marks]

4. Let $(\mathbf{i}, \mathbf{j}, \mathbf{k})$ be a set of mutually orthogonal unit vectors. Suppose that $\mathbf{a} = \mathbf{i} + \mathbf{j} - 2\mathbf{k}$ and $\mathbf{b} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$. Find

   (a) $2\mathbf{a} - 3\mathbf{b}$, (b) $|\mathbf{b}|$, (c) $\mathbf{a}.\mathbf{b}$, (d) a unit vector parallel to $\mathbf{a} - \mathbf{b}$.

   Find also
   (e) the angle between $\mathbf{a}$ and $\mathbf{b}$ correct to the nearest degree.

   [17 marks]

5. Let $(\mathbf{i}, \mathbf{j}, \mathbf{k})$ be a right handed set of mutually orthogonal unit vectors. Suppose that $\mathbf{a} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$ and $\mathbf{b} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$. Calculate $\mathbf{a} \times \mathbf{b}$.

   [6 marks]

6. Evaluate the determinant

$$\begin{vmatrix} 2 & 3 & -5 \\ 2 & -1 & 3 \\ 1 & -1 & -1 \end{vmatrix}.$$

[9 marks]

SECTION B

7. (a) Let A and B be the points with position vectors $\mathbf{a}$ and $\mathbf{b}$, respectively. The line l has vector equation

$$\mathbf{r} = (1 - \lambda)\mathbf{a} + \lambda\mathbf{b}.$$

Explain the relation of l to A and B.
Draw a sketch showing a point given by a negative value of λ?

(b) Let A, B, C and D be the points $(2, -3, -1)$, $(3, 0, 1)$, $(7, 4, 11)$ and $(0, -1, -7)$, respectively. Show that the line through A and B meets the line through C and D, and find the point of intersection X.

[15 marks]

8. Let O be a fixed origin and $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ be constant mutually orthogonal unit vectors.

(a) The position vector with respect to O of a particle P is

$$\mathbf{r}(t) = \{3\mathbf{i} + (t - 1)\mathbf{j} + (7t - 2)\mathbf{k}\} \text{ metres}$$

at time t seconds. Find

(i) the position vector of P at time $t = 0$;

(ii) the velocity of P at time t seconds.

Describe the path of the particle.

(b) Let $\mathbf{a} = (\cos t)\mathbf{i} + (\sin t)\mathbf{j}$. Show that $\mathrm{d}\mathbf{a}/\mathrm{d}t$ is orthogonal to $\mathbf{a}$ and that both $\mathbf{a}$ and $\mathrm{d}\mathbf{a}/\mathrm{d}t$ are unit vectors.

[15 marks]

9. The points A, B and C have Cartesian coordinates $(5, 3, -1)$, $(7, 4, 1)$ and $(7, 3, 0)$, respectively. Find

   (a) the angles of the triangle ABC, correct to the nearest degree;

   (b) a unit vector perpendicular to both AB and AC;

   (c) the Cartesian equation of the plane Π through A, B and C.

[**15 marks**]

10. Use the method of elimination to find the solution of the simultaneous equations

$$\begin{array}{rcrcrcrcl} w & + & 3x & - & 9y & - & 4z & = & 1 \\ w & + & 4x & - & 4y & - & 3z & = & 1 \\ 2w & + & x & - & 8y & - & 3z & = & 7 \\ w & + & 5x & + & 2y & - & z & = & -1\,. \end{array}$$

[**15 marks**]