TIME ALLOWED: Three Hours

INSTRUCTIONS TO CANDIDATES

Answer ALL questions in Section A and THREE questions from Section B. The total of the marks available on Section A is 55.

SECTION A

1. Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

   (a) Express $\overrightarrow{AB}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

   (b) Let X be the point on AB such that $\overrightarrow{AX} = 2\overrightarrow{XB}$. Find $\overrightarrow{OX}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

[8 marks]

2. Let $(\mathbf{i}, \mathbf{j}, \mathbf{k})$ be a right handed set of mutually orthogonal unit vectors. Suppose that $\mathbf{a} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $\mathbf{b} = 3\mathbf{i} - \mathbf{k}$. Find

   (a) $2\mathbf{a} - 3\mathbf{b}$, (b) $|\mathbf{a}|$, (c) $\mathbf{a}.\mathbf{b}$, (d) $\mathbf{a} \times \mathbf{b}$.

   Find also
   (e) the angle between $\mathbf{a}$ and $\mathbf{b}$ correct to the nearest degree,
   and
   (f) a unit vector orthogonal to both $\mathbf{a}$ and $\mathbf{b}$.

[18 marks]

3. Let O be a fixed origin and let $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ be constant mutually orthogonal unit vectors. The position vector with respect to O of a particle P is

$$\mathbf{r}(t) = \{5\mathbf{i} + (4t + 3)\mathbf{j} + (7t - t^2)\mathbf{k}\}\text{metres.}$$

at time t seconds. Find

(a) the position of P at time $t = 0$;

(b) the velocity of P at time t seconds;

(c) the speed of P when $t = 2$;

(d) the acceleration of P at time t seconds.

[14 marks]

4. Evaluate the determinant

$$\begin{vmatrix} 3 & 5 & 7 \\ 2 & -1 & 3 \\ 2 & 1 & 2 \end{vmatrix}.$$

[9 marks]

5. Let $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ be unit vectors parallel to the coordinate axes Ox, Oy and Oz. Suppose that $\mathbf{n} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$. Find the Cartesian equation of the plane through the point $(1, 0, -1)$ and perpendicular to $\mathbf{n}$.

[6 marks]

SECTION B

6. Suppose that A, B, C and D are four distnct, non-collinear points in space and that $\overrightarrow{AB} = \mathbf{x}$, $\overrightarrow{BC} = \mathbf{y}$ and $\overrightarrow{CD} = \mathbf{z}$.

   (a) Find an expression for $\overrightarrow{DA}$ in terms of $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{z}$.

   (b) What condition must be satisfied by $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{z}$ in order that $ABCD$ should be a parallelogram?

   (c) Suppose that, in terms of mutually orthogonal unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$,

   $$\mathbf{x} = 2\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}, \qquad \mathbf{y} = -2\mathbf{j} + \mathbf{k}, \qquad \text{and} \qquad \mathbf{z} = 2\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}.$$

   Show that

   i. $ABCD$ is not a parallelogram;

   ii. A, B, C and D lie in the same plane.

[15 marks]

7. The points A, B and C have Cartesian coordinates $(2, 1, 0)$, $(1, -1, -1)$ and $(-1, -1, 1)$, respectively. Find

   (a) the area of the triangle ABC;

   (b) the Cartesian equation of the plane Π through A, B and C;

   (c) the distance from the plane Π of the point X with coordinates $(1, 2, 3)$.

[15 marks]

8. (a) Find a parametric equation for the line of intersection of the planes

$$x - y + 3z = 3 \qquad \text{and} \qquad 3x + y - z = 11.$$

(b) Let A, B and C be the points with Cartesian coordinates $(3, -1, 1)$, $(1, -3, 3)$ and $(1, -1, 2)$, respectively. Find the volume of the parallelepiped with edges parallel to OA, OB and OC and which has O, A, B, and C as four of its vertices.

[15 marks]

9. Use the method of elimination to find the solution of the simultaneous equations

$$\begin{array}{rcrcrcrcr} x & + & 2y & - & z & - & 3w & = & 2 \\ x & + & 3y & + & 4z & + & 4w & = & 5 \\ 2x & + & 3y & - & 6z & + & 3w & = & -13 \\ x & + & 2y & & & + & 4w & = & -3\,. \end{array}$$

[15 marks]