1. (a) $\dfrac{(a^3b)^5c^{-4}}{a^{15}b^4c^3} = \dfrac{a^{15}b^5c^{-4}}{a^{15}b^4c^3} = \dfrac{b}{c^7}$ **[2]**

(b) $\dfrac{1-16x^2}{1+8x+16x^2} = \dfrac{(1-4x)(1+4x)}{(1+4x)^2} = \dfrac{1-4x}{1+4x}$ **[2]**

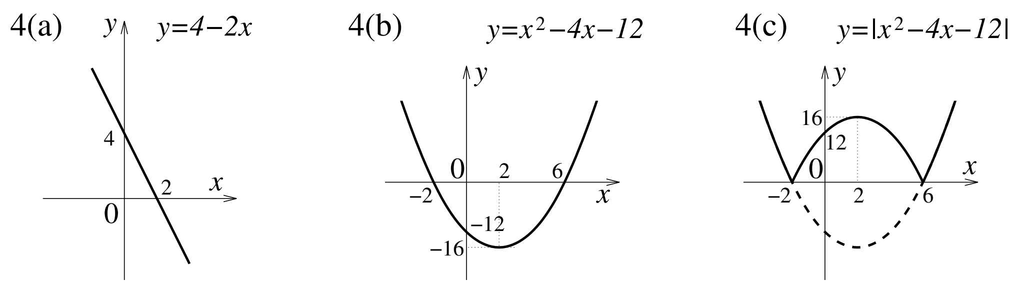
2. $\dfrac{2}{2x+5} + \dfrac{5}{2x^2+5x} = \dfrac{2}{2x+5} + \dfrac{5}{x(2x+5)} = \dfrac{2x+5}{x(2x+5)} = \dfrac{1}{x}$ **[4]**

3. (a) $x^2+3x-28 = (x-4)(x+7)$ so solutions are $x = 4, x = -7$. **[2]**

(b) Using the quadratic formula

$$x = \frac{-13 \pm \sqrt{169 - 4 \times 12 \times (-35)}}{24} = \frac{-13 \pm 43}{24} = \frac{5}{4} \quad \text{or} \quad -\frac{7}{3}$$ **[2]**

4. (a) $y = -2x+4$ represents a straight line with slope -2 meeting the y-axis at $y = 4$. **[2]**
(b) $y = x^2-4x-12$ is a quadratic curve, which is U-shaped, crossing the y-axis at $y = -12$ and the x-axis at $x = -2, x = 6$. The curve is symmetric about the line $x = -(-4)/2 = 2$. The vertex is at $x = 2, y = -16$. **[3]**
(c) $y = |x^2-4x-12|$ is obtained from (b) by reflecting the part below the x-axis in the x-axis. **[2]**



5. Put $y = \dfrac{3-2x}{4x+5}$, and solve for x in terms of $y = f(x)$. Then $(4x+5)y = 3-2x$ so $4xy+5y = 3-2x$, giving $x(4y+2) = 3-5y$.
Thus $x = f^{-1}(y) = \dfrac{3-5y}{4y+2}$ and so $f^{-1}(x) = \dfrac{3-5x}{4x+2}$. **[3]**

6. (a) Either use the formula $S_n = a\dfrac{1-r^n}{1-r}$ with $r = -5, a = -5, n = 4$, giving $-5 \times \dfrac{(-5)^4-1}{-5-1} = -520$ or simply add up the 4 terms. **[3]**

(b) The formula is $\dfrac{a}{1-r}$. [1]

Here $a = \frac{2}{11}, r = \frac{2}{11}$, giving the sum as $\dfrac{\frac{2}{11}}{1-\frac{2}{11}} = \dfrac{\frac{2}{11}}{\frac{9}{11}} = \dfrac{2}{9}$. [2]

7. **(a)** $\displaystyle\lim_{n\to\infty} \frac{4n^2-7}{9-3n-2n^2} = \lim_{n\to\infty} \frac{4-\frac{7}{n^2}}{\frac{9}{n^2}-\frac{3}{n}-2} \to \frac{4}{-2} = -2$ as $n \to \infty$. [2]

(b) Putting $x = 6$ in the bottom of the fraction gives 0, as also in the top. Factorise to write

$$\frac{x^2-36}{x^2-2x-24} = \frac{(x-6)(x+6)}{(x-6)(x+4)} = \frac{x+6}{x+4}$$

Now put $x = 6$ to get the limit $\frac{12}{10} = \frac{6}{5}$. [2]

8. **(a)** Put $u = 5x - 6$. Then $y = (5x-6)^4 = u^4$ so

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = 4u^3 \times 5 = 20(5x-6)^3. \qquad [2]$$

(b) Put $u = x^7 + 2$. Then $y = (x^7+2)^{6/7} = u^{6/7}$ and

$$\frac{dy}{dx} = \frac{6}{7}u^{-1/7} \times \frac{du}{dx} = \frac{6}{7}u^{-1/7} \times 7x^6 = 6x^6(x^7+2)^{-1/7}. \qquad [3]$$

(c) $y = x^3 \cos x = uv$ with $u = x^3, v = \cos x$.

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx} = -x^3 \sin x + 3x^2 \cos x. \qquad [3]$$

9. The slope of the tangent is the value of $\dfrac{dy}{dx}$ at $x = 2$. Now $\dfrac{dy}{dx} = -3x^2$, so the slope is -12. When $x = 2$ we have $y = -4$, so the tangent line has equation $y + 4 = -12(x-2)$, giving $y = -12x + 20$. [3]

10. **(a)** $\displaystyle\int (\sin x + 3x^5 - 4)\, dx = \int \sin x\, dx + 3\int x^5\, dx - 4\int 1\, dx = -\cos x + \frac{1}{2}x^6 - 4x + C$ [4]

(b) $\displaystyle\int e^{-4x}\, dx = \frac{1}{-4}e^{-4x} + C = -\frac{1}{4}e^{-4x} + C.$ [2]

11. **(a)** $\displaystyle\int_0^{\frac{\pi}{10}} \cos 5x\, dx = \left[\frac{1}{5}\sin 5x\right]_0^{\frac{\pi}{10}} = \frac{1}{5}\sin\frac{\pi}{2} - \frac{1}{5}\sin 0 = \frac{1}{5}.$ [3]

(b) Substitute $u = 6x + 1$ so that $du = 6dx$. Then

$$I = \int_0^4 \frac{6}{6x+1}\,dx = \int_{x=0}^{x=4} \frac{1}{u}\,du = \int_{u=1}^{u=25} \frac{1}{u}\,du = [\ln u]_1^{25} = \ln 25 - \ln 1 = \ln 25 \qquad [3]$$

12. (i) Differentiate the LHS to get

$$2 \times 3x^2 + 3(x \times 2y\frac{dy}{dx} + y^2) - 7 \times 3y^2\frac{dy}{dx} \qquad [4]$$

The RHS has derivative 0 giving the equation $6x^2 + 6xy\frac{dy}{dx} + 3y^2 - 21y^2\frac{dy}{dx} = 0$. [1]

Then $3(7y^2 - 2xy)\frac{dy}{dx} = 3(2x^2 + y^2)$ so $\frac{dy}{dx} = \frac{2x^2 + y^2}{7y^2 - 2xy}$. [2]

(ii) The slope of the tangent line when $x = 2, y = 1$ is then $\frac{8+1}{7-4} = 3$ and the line has equation $y - 1 = 3(x - 2)$, so that $y = 3x - 5$. [4]

(iii) If this line meets the curve $y = x^2 + x - 4$ at a point with the horizontal coordinate x, then $x^2 + x - 4 = 3x - 5$, giving the quadratic equation $x^2 - 2x + 1 = 0$. The only solution is $x = 1$ so the line and the curve meet in exactly one point. [4]

13. (a) Put $u = 3 - \sin x$. Then $y = \ln(3 - \sin x) = \ln u$, so

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{1}{u} \times (-\cos x) = \frac{\cos x}{\sin x - 3} \qquad [3]$$

(b) $y = e^{4x+5}(2 - 3x) = uv$ with $u = e^{4x+5}, v = 2 - 3x$.

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx} = e^{4x+5} \times (-3) + (2 - 3x) \times 4e^{4x+5} = e^{4x+5}(5 - 12x). \qquad [4]$$

(c) In $u = \cos^6 x$, put $w = \cos x$. Then $u = w^6$. Hence $\frac{du}{dx} = \frac{du}{dw} \times \frac{dw}{dx} = 6w^5 \times (-\sin x) = -6\cos^5 x\ \sin x$. Therefore $\frac{d}{dx}(\cos^6 x - 5x^7 + 8) = -6\cos^5 x\ \sin x - 35x^6$. [4]

(d) Put $v = \sin^3 x$ and $w = \sin x$. Then $v = w^3$ and $\frac{du}{dx} = \frac{dv}{dw} \times \frac{dw}{dx} = 3w^2 \times \cos x = 3\sin^2 x\ \cos x$. Setting also $u = 4 - 2x^5$, we have

$$\frac{d}{dx}\frac{4 - 2x^5}{\sin^3 x} = \frac{d}{dx}\frac{u}{v} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

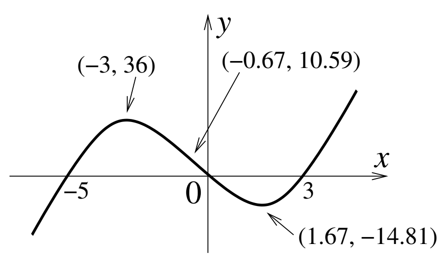
$$= \frac{\sin^3 x \times (-10x^4) - (4 - 2x^5) \times 3\sin^2 x\ \cos x}{\sin^6 x} = -\frac{10x^4 \sin x + 3(4 - 2x^5)\cos x}{\sin^4 x} \qquad [4]$$

14. (i) The stationary points occur where $f'(x) = 0$ and the inflection points where $f''(x) = 0$. Now $f'(x) = 3x^2 + 4x - 15$ and $f''(x) = 6x + 4$. [3]

The inflection point occurs where $x = -\frac{2}{3} = -0.67$ and $f(x) = 10 + \frac{16}{27} = 10.59$. **[1]**

The stationary points are given by the quadratic formula as $x = \dfrac{-4 \pm \sqrt{16+180}}{6} = \dfrac{-2 \pm 7}{3} = 1.67$ or -3. These are respectively a local minimum, where $f''(x) > 0$, and a local maximum, where $f''(x) < 0$.
The corresponding values of $f(x)$ are -14.81 and 36. **[2]**
(ii) The curve $y = x^3 + 2x^2 - 15x$ crosses the x-axis when $x^3 + 2x^2 - 15x = 0$. This happens when $x = 0$ or when $x^2 + 2x - 15 = 0$, giving $x = -5$ or $x = 3$ as well. **[2]**
(iii) Using the information from (a) and (b), sketch the curve $y = x^3 - 2x^2 - 8x$. **[3]**
(iv) The total area bounded by the curve and the x-axis is made up of two pieces, one between $x = -5$ and $x = 0$, and the other between $x = 0$ and $x = 3$. These are found as $\left|\int_{-5}^{0} f(x)\,dx\right|$ and $\left|\int_{0}^{3} f(x)\,dx\right|$. Now $\int f(x)\,dx = \int (x^3 + 2x^2 - 15x)\,dx = \dfrac{x^4}{4} + \dfrac{2}{3}x^3 - \dfrac{15}{2}x^2$, giving the first area as $|-625/4 + 250/3 + 375/2| = 1375/12$ and the second as $|81/4 + 18 - 135/2| = 119/4$ making a total of $1375/12 + 119/4 = 433/3 = 144.33$. **[4]**

14(iii)



15. (a) Substitute $u = x^6 + 8$. Then $du = 6x^5 dx$, so

$$I = \int x^5 \cos(x^6 + 8)\,dx = \int \frac{1}{6} \cos u\,du = \frac{1}{6} \sin u + C = \frac{1}{6} \sin(x^6 + 8) + C. \qquad [4]$$

(b) Substitute $t = \tan x$. Then $dt = \sec^2 x\,dx$, so

$$I = \int \tan^9 x\ \sec^2 x\ dx = \int t^9\,dt = \frac{t^{10}}{10} + C = \frac{1}{10} \tan^{10} x + C. \qquad [4]$$

(c) Substitute $x = \frac{4}{5} \sin t$. Then $dx = \frac{4}{5} \cos t\,dt$. Therefore

$$I = \int_0^{2\sqrt{2}/5} \frac{dx}{\sqrt{16 - 25x^2}} = \int_{x=0}^{x=2\sqrt{2}/5} \frac{\frac{4}{5}\cos t}{\sqrt{16 - 16\sin^2 t}}\,dt =$$

$$= \int_{x=0}^{x=2\sqrt{2}/5} \frac{\frac{4}{5}\cos t}{\sqrt{16\cos^2 t}}\,dt = \frac{1}{5} \int_{x=0}^{x=2\sqrt{2}/5} dt = \frac{1}{5}[t]_{x=0}^{x=2\sqrt{2}/5}$$

Now when $x = 0$ we have $t = 0$, and when $x = 2\sqrt{2}/5$ we have $2\sqrt{2}/5 = \frac{4}{5}\sin t$ so that $\sin t = \frac{\sqrt{2}}{2}$ and $t = \frac{\pi}{4}$. Consequently $I = \frac{1}{5}[t]_{t=0}^{t=\pi/4} = \frac{1}{5}(\frac{\pi}{4} - 0) = \frac{\pi}{20} = 0.16$. **[7]**