Candidates should answer the WHOLE of Section A and THREE questions from Section B. Section A carries 55% of the available marks.

SECTION A

1. Simplify

$$(i)\ \frac{a^2b^3c^5}{ab^2c^3}\ ; \qquad (ii)\ \frac{b^2-c^2}{c(b+c)}\ ; \qquad (iii)\ \frac{b^2-2b-3}{b^2+3b+2}\ .$$

[5 marks]

2. Sketch the graph of each of the functions

$$(i)\ y = 3x-1\ ; \qquad (ii)\ y = |3x-1|\ ; \quad (iii)\ y = (x-1)^3 .$$

[6 marks]

3. State, giving your reasons, whether the following functions are even, odd or neither even nor odd:

$$(i)\ f(x) = x^4+5x^2-7\ ; \qquad (ii)\ f(x) = \frac{1-x^2}{1+x}\ .$$

[4 marks]

4. Determine the values of x for which

$$x^2+6x+8 \leq 0\ .$$

[3 marks]

5. Given that $y = f(x) = 2x-1$, obtain an expression for the inverse function $f^{-1}(x)$. [2 marks]

6. Find the sum of each of the following geometric series (giving your answer as a rational number):

$$(i)\ \sum_{n=1}^{8} 3^{n-1}\ ; \qquad (ii)\ \sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n .$$

[4 marks]

7. Evaluate the following limits

$$(i)\ \lim_{n\to\infty} \frac{20n^2+5n-2}{4n^2-6n+1}\ ; \qquad (ii)\ \lim_{x\to 4} \frac{x^2-16}{x-4}\ .$$

[4 marks]

8. Simplify

$$\frac{1}{x+h} - \frac{1}{x} \ .$$

Hence, or otherwise, differentiate *from first principles*, the function

$$y = x^{-1} \ .$$

[4 marks]

9. Differentiate with respect to x

$$(i)\ 4x^3 \ ; \qquad (ii)\ (x^2+1)^2 \ ; \qquad (iii)\ x^3 \sin(3x) \ .$$

[5 marks]

10. Given that $2x^2y + 3xy^3 = 14$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $x = 2,\ y = 1$. [4 marks]

11. Find and classify all stationary points of the function

$$f(x) = x + \frac{4}{x} \ .$$

[5 marks]

12. Find the following indefinite integrals

$$(i) \int (2x - \cos x)\ \mathrm{d}x; \qquad (ii) \int \mathrm{e}^{\frac{1}{2}x+1}\ \mathrm{d}x.$$

[4 marks]

13. Evaluate the following definite integrals

$$(i) \int_0^2 2(x+2)\ \mathrm{d}x; \qquad (ii) \int_{\pi/12}^{\pi/4} \cos(2x)\ \mathrm{d}x.$$

[5 marks]

SECTION B

14. The function f is periodic with period 2 and is defined to be

$$f(x) = 2x - x^2 \ , \qquad \text{for } 0 \leq x \leq 2 \ ,$$

and

$$f(x+2) = f(x).$$

Determine the values of $f(-1)$, $f(1)$, $f(\frac{3}{2})$ and $f(20)$.
Sketch the graph of $f(x)$ for $-2 \leq x \leq 8$.
Find the slope of the tangent to $y = f(x)$ at $x = \frac{3}{2}$. Find the equation of this tangent. For what value of x does this tangent meet the x–axis? [15 marks]

15. Find where the cubic

$$y = f(x) = (x-2)(x^2 - 4x + 1)$$

crosses the x–axis. Give non-integer values to 1 decimal place.
By expanding the brackets, or otherwise, find $f'(x)$ and $f''(x)$.
Hence find the two stationary points and determine their nature.
Find the point of inflection.
Sketch the graph of $f(x)$.

[15 marks]

16 (a). Differentiate the following functions with respect to x:

$$(i) \ \frac{x^2+1}{x-1} \ ; \qquad (ii) \ \mathrm{e}^{-3x}\cos(3x) \ .$$

[7 marks]

(b.) Find the following indefinite integrals:

$$(i) \int x^2 \mathrm{e}^{x^3} \mathrm{d}x \ ; \qquad (ii) \int x \sin(3x) \mathrm{d}x$$

[8 marks]

17 (a). Evaluate the following definite integrals:

$$(i) \int_0^1 3x\mathrm{e}^{3x} \mathrm{d}x \ ; \qquad (ii) \int_0^2 \frac{4x}{x^2+1} \mathrm{d}x \ .$$

[8 marks]

(b). Find the area of the region enclosed by the curves $y = 8 - x^2$ and $y = x^2$. [7 marks]