Candidates should answer the WHOLE of Section A and THREE questions from Section B. Section A carries 55% of the available marks.

JANUARY 2001

SECTION A

1. Simplify

$$(i)\ \frac{a^3b^2c^2}{a^5bc^4}\ ; \qquad\qquad (ii)\ \frac{ac+bc}{a^2-b^2}\ .$$

[3 marks]

2. Given that

$$\frac{3}{x}+\frac{2}{y}=\frac{1}{z}\ ,$$

find an explicit expression for x in terms of y and z. [3 marks]

3. A straight line passes through the points $x = 1,\ y = 2$ and $x = 3,\ y = 8$. Find the slope of the line and its intercept in the $y-$axis. [2 marks]

4. Sketch the graph of each of the functions

$$(i)\ y = 3 - x\ ; \qquad (ii)\ y = x^2 - 2\ ; \quad (iii)\ y = |x^2 - 2|\ .$$

[5 marks]

5. Determine the values of x for which

$$x^2 - x - 6 < 0\ .$$

[3 marks]

6. Determine which of the following functions are even, odd or neither even nor odd:

$$(i)\ x + x^3\ ; \qquad (ii)\ x^2 \cos x\ , \qquad (iii)\ 1 + x + x^2\ .$$

[3 marks]

7. Given that $y = f(x) = 3x + 7$, obtain an expression for the inverse function $f^{-1}(x)$. [2 marks]

8. Find the sum of the geometric series

$$(i)\ 4\sum_{n=1}^{4}\left(\frac{1}{4}\right)^{n}; \qquad (ii)\ \sum_{n=1}^{\infty}\left(\frac{2}{7}\right)^{n}.$$

[4 marks]

9. Evaluate the following limits

$$(i)\ \lim_{n\to\infty}\frac{1+2n+3n^2}{3-4n+5n^2}; \qquad (ii)\ \lim_{x\to 4}\frac{x^2-2x-8}{x^2-16}.$$

[4 marks]

10. Differentiate *from first principles*, the function

$$y = 2x^2 .$$

[3 marks]

11. Differentiate with respect to x

$$(i)\ 7x^5; \qquad (ii)\ (3x-2)^4; \qquad (iii)\ x^3\cos(4x) .$$

[5 marks]

12. Given that $x^3+3xy^2-4y^3=10$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $x=1,\ y=2$. [3 marks]

13. Find and classify all stationary points of the function

$$f(x) = 4x^2 + \frac{1}{x} .$$

[6 marks]

14. Find the indefinite integrals

$$(i)\ \int (\sin x - 3x^2)\ \mathrm{d}x; \qquad (ii)\ \int \mathrm{e}^{-3x}\ \mathrm{d}x.$$

[4 marks]

15. Evaluate the definite integrals

$$(i)\ \int_0^1 \frac{4}{2x+1}\ \mathrm{d}x; \qquad (ii)\ \int_0^{\pi/2} 6\cos(3x)\ \mathrm{d}x.$$

[4 marks]

SECTION B

16. The function $f(x)$ has period 2 and is defined to be

$$f(x) = x^2 - 4x + 3 \qquad 1 \le x \le 3$$
$$f(x+2) = f(x) \qquad \text{for all } x.$$

Determine the values of $f(2)$ and $f(\frac{5}{2})$. Find the slope of the tangent to $y = f(x)$ at $x = \frac{5}{2}$. Determine also the values of $f(0)$ and $f(\frac{1}{2})$ and sketch the graph of $y = f(x)$ for $-1 \le x \le 5$.
Find the value of x where the tangent to this curve at $x = \frac{1}{2}$ meets the $x-$axis.
[15 marks]

17 (a). Differentiate the following

$$(i)\ \frac{x-1}{x^2+x+1}\ ; \qquad (ii)\ \mathrm{e}^{2x}\cos(3x+2)\ .$$

(b.) Find the following indefinite integrals

$$(i)\ \int x\cos(x^2+1)\ \mathrm{d}x\ ; \qquad (ii)\ \int x\mathrm{e}^{3x}\ \mathrm{d}x\ .$$

[15 marks]

18. The function f is defined to be

$$f(x) = \frac{4}{x+2} - \frac{1}{x-2}\ ,$$

find f' and f''. Show that $x = 6$, $x = \frac{2}{3}$ are stationary points of f and determine their nature. Find the equations of the one horizontal and two vertical asymptotes to the graph of $y = f(x)$. Hence sketch the graph. [15 marks]

19 (a). Evaluate the definite integrals:

$$(i)\ \int_0^{\pi/2} x\sin x\ \mathrm{d}x\ ; \qquad (ii)\ \int_1^2 \frac{6x^2}{x^3+1}\ \mathrm{d}x\ .$$

(b). Find the three points where the graph of the curve $y = x^3 - x^2 - 2x$ crosses the $x-$axis. Calculate the *total* area bounded by the curve and the $x-$axis.
[15 marks]