Candidates should answer the WHOLE of Section A and THREE questions from Section B. Section A carries 55% of the available marks.

JANUARY 2000

SECTION A

1. Simplify

$$(i)\ \frac{a^3b^2c}{ab^3c^2}\ ; \qquad (ii)\ \frac{a+b}{b^2-a^2}\ .$$

[2 marks]

2. Given that

$$\frac{2}{a}-\frac{3}{b}=\frac{3}{c}\ ,$$

find an explicit expression for a in terms of b and c. [3 marks]

3. Find the equation of the straight line that has gradient 2 and passes through the point (1,3). [2 marks]

4. Sketch the graph of each of the functions

$$(i)\ y=3x-2\ ; \qquad (ii)\ y=1-x^2\ ; \quad (iii)\ y=|x-4|\ .$$

[5 marks]

5. Determine the values of x for which

$$(x+2)^2>9x-2\ .$$

[4 marks]

6. In each of the following cases, determine whether the function is even, odd or neither even nor odd:

$$(i)\ 1-2x^2\ ; \qquad (ii)\ x+\sin x\ , \qquad (iii)\ x^3+1\ .$$

[4 marks]

7. Given that $y=f(x)=3-5x$, obtain an expression for the inverse function $f^{-1}(x)$. [2 marks]

8. Find the sum of each of the series

$$(i)\ \sum_{n=1}^{5}\left(\frac{1}{2}\right)^n\ ; \qquad (ii)\ \sum_{n=1}^{\infty}\left(\frac{3}{5}\right)^n\ .$$

[4 marks]

9. Evaluate the following limits

$$(i)\ \lim_{n\to\infty} \frac{n^2+4}{3n^2+5n+2}\ ; \qquad (ii)\ \lim_{x\to 3} \frac{3x-x^2}{x-3}\ .$$

[4 marks]

10. Differentiate *from first principles*, the function

$$y = x^2\ .$$

[3 marks]

11. Differentiate with respect to x

$$(i)\ 5x^3\ ; \qquad (ii)\ (3-2x)^3\ ; \qquad (iii)\ x^3\sin(2x)\ .$$

[5 marks]

12. Given that $x^2+3xy+2y^2=15$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point (1,2). [3 marks]

13. Find the stationary points of the function

$$f(x) = x + \frac{4}{x+1}$$

and determine their nature. [6 marks]

14. Find the indefinite integrals

$$(i)\ \int (x^3+\mathrm{e}^{-2x})\ \mathrm{d}x; \qquad (ii)\ \int x^{-3}\ \mathrm{d}x.$$

[4 marks]

15. Evaluate the definite integrals:

$$(i)\ \int_0^{\frac{\pi}{3}} \sin(2x)\ \mathrm{d}x; \qquad (ii)\ \int_2^3 \frac{3}{x-1}\ \mathrm{d}x.$$

[4 marks]

SECTION B

16 (a). The function $f(x)$ is periodic with period 5 and

$$f(x) = x(x-5) \qquad \text{for } 0 \le x < 5 \; .$$

Determine $f(2)$, $f(7)$ and $f(14)$. Sketch the graph of $f(x)$ for $-5 \le x \le 15$.

[10 marks]

(b.) Find the equation of the tangent to the curve

$$y = x\mathrm{e}^x$$

at the point (1,e). Determine where this tangent meets the coordinate axes.

[5 marks]

17 (a). Differentiate the following functions with respect to x:

$$(i)\ (x^2+1)^2\cos(x^2+1)\ ; \qquad (ii)\ \frac{\sin x}{x^4+1}\ .$$

[7 marks]

(b.) Find the following indefinite integrals

$$(i)\ \int x\mathrm{e}^{2x}\ \mathrm{d}x\ ; \qquad (ii)\ \int x(x^2+2)^3\ \mathrm{d}x$$

[8 marks]

18. The function $f(x)$ is defined to be

$$y = f(x) = 1 + \frac{1}{x^2-4}\ .$$

Find the first and second derivatives $f'(x)$ and $f''(x)$. Show that the function $f(x)$ has one stationary point and determine its nature. Find the equations of the one horizontal and two vertical asymptotes to the graph of $y = f(x)$. Hence sketch the graph of $f(x)$. [15 marks]

19 (a). Evaluate the definite integrals:

$$(i)\ \int_1^2 (x+x^{-1})^2\ \mathrm{d}x\ ; \qquad (ii)\ \int_0^1 x\sin(\pi x)\ \mathrm{d}x\ .$$

[10 marks]

(b). Sketch the graph of the curve $y = x^2 - x - 2$. Find the area of the finite region enclosed by this curve and the $x-$axis. [5 marks]