SECTION A

1. Given $e = 1.602177 \times 10^{-19}$ C and $\epsilon_0 = 8.854185 \times 10^{-12}$ F m^{-1}, express $e^2/(4\pi\epsilon_0)$ in standard form, accurate to 4 significant figures.

[4 marks]

2. Simplify

(i)
$$\frac{(x^3 z)^{\frac{1}{2}}}{(xy^{-1})^3 (yz^{-2})^{\frac{1}{4}}},$$

(ii)
$$\frac{1}{2+\sqrt{3}} + \frac{1}{2-\sqrt{3}}.$$

[6 marks]

3. Evaluate

(i) $\tan^{-1}(0.3572)$ (in radians to 3 decimal places),

(ii)
$$\binom{7}{4}.$$

[4 marks]

4. A triangle has sides of lengths 7 cm, 6 cm and 4 cm.
Find **one** of the angles of this triangle in degrees to 4 significant figures.
[Hint: The cosine rule is $c^2 = a^2 + b^2 - 2ab\cos C$.] [3 marks]

5. Given that

$$S = \sum_{i=0}^{N} u_i \equiv 9 + 13 + 17 + \cdots + 89,$$

write down the values of u_0 and N and the expression for u_i. Evaluate S.

[4 marks]

6. Evaluate the sum of the geometric series

$$\frac{1}{6}-\frac{1}{18}+\frac{1}{54}-\cdots-\frac{1}{13122}$$

to 5 decimal places. [4 marks]

7. Sketch

$$y=\frac{x-2}{x-1}.$$

[4 marks]

8. Sketch $f(\theta)=2\sin(\theta+\frac{\pi}{3})$ for $-\pi\le\theta\le\pi$.
Is $f(\theta)$ (i) an even function, (ii) an odd function or (iii) neither? [4 marks]

9. Find the roots of the equation $10x^2-9x-7=0$.
Hence factorise $f(x)=10x^2-9x-7$. [4 marks]

10. Differentiate $x=(2t)^2\mathrm{e}^{-4t}$ with respect to t. [3 marks]

11. Find the area of the region bounded by the lines
$x=1$, $x=2$, $y=0$ and $y=2\mathrm{e}^{2x}$. [5 marks]

12. Find all the second order partial derivatives of $\phi(x,y)=(x^2+y^2)^{\frac{1}{2}}$.
[5 marks]

13. Express each of the complex numbers in modulus-argument form

$$z_1=3+4j, \qquad z_2=12-5j.$$

Hence, or otherwise, find (i) $|z_2/z_1|$ and
(ii) $\arg(z_1z_2)$ (in radians to 2 decimal places). [5 marks]

SECTION B

14 (a). Using Pascal's triangle, or otherwise, expand

$$(a+b)^5.$$

Check your answer for the case $a = 1$, $b = -\frac{1}{2}$. [8 marks]

(b). Find the first and the second derivatives of $I = A\cos 2t + B\sin 2t$ with respect to t.

Given that

$$\frac{\mathrm{d}^2 I}{\mathrm{d}t^2} + 2\frac{\mathrm{d}I}{\mathrm{d}t} + 3I = 10\cos 2t - 11\sin 2t$$

for all t, show that

$$-A + 4B = 10, \qquad 4A + B = 11.$$

Hence, determine $I(t)$. [7 marks]

15 (a). Evaluate the integrals

$$\int_0^{\frac{\pi}{2}} \theta^2 \sin\theta \ \mathrm{d}\theta.$$

[8 marks]

(b). <u>*Either*</u> Give one example of an improper integral.
Evaluate your integral.

<u>*or*</u> The function $\ln(x)$ is defined by

$$\ln(x) = \int_1^x \frac{1}{t}\mathrm{d}t.$$

Using the properties of definite integrals, show that

(i) $\ln(ax) = \ln(a) + \ln(x)$

(ii) $\ln(\mathrm{e}^x) = x$. [Hint: consider $t = \mathrm{e}^s$.] [7 marks]

16. Given the cubic $y = \mathrm{f}(x) = x^3 - 9x^2 + 24x - 20$,
evaluate f(1), f(4) and f(5).
Hence, or otherwise, find the factors of $\mathrm{f}(x)$.
Find and classify the stationary points of $\mathrm{f}(x)$.
Sketch $\mathrm{f}(x)$. [15 marks]

17 (a). Use De Moivre's theorem to show that

$$\sin 4\theta = 4\sin\theta\cos\theta(2\cos^2\theta - 1).$$

[8 marks]

(b). Write down the value of $\mathrm{e}^{2jk\pi}$, where k is an integer.

Find the complex numbers z for which $z^4 = 16j$. Give your answers in the form $r\mathrm{e}^{j\theta}$.

Show these numbers as points on the Argand diagram. [7 marks]

18 (a). Evaluate the double integrals

$$\int\int_{\mathcal{R}} \rho(x,y)\mathrm{d}x\mathrm{d}y, \qquad \int\int_{\mathcal{R}} x\rho(x,y)\mathrm{d}x\mathrm{d}y, \qquad \int\int_{\mathcal{R}} y\rho(x,y)\mathrm{d}x\mathrm{d}y$$

over the lamina $\mathcal{R}$ $(0 \le x \le 1-y,\ 0 \le y \le 1)$ when $\rho(x,y) = x^2 + y^2$ Kg m^{-2}.

Hence, find the coordinates of the centre of mass of the lamina $\mathcal{R}$.

[9 marks]

(b). Given that

$$x = X\cos\alpha - Y\sin\alpha, \qquad y = X\sin\alpha + Y\cos\alpha,$$

where α is a constant, use the chain rule to find $\frac{\partial z}{\partial X}$ and $\frac{\partial z}{\partial Y}$ in terms of $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

Comment, with the aid of a diagram, on your result. [6 marks]