Instructions to candidates

Candidates should answer the WHOLE of Section A and THREE questions from Section B. Section A carries 55% of the available marks.

SECTION A

1. Given $h = 6.6262 \times 10^{-34}$ J s, $c = 2.9979 \times 10^{8}$ ms^{-1} and $\hbar = h/2\pi$, express $(\hbar c)^3$ in standard form, accurate to 4 significant figures. [3 marks]

2. Simplify the following without using a calculator. Show your working.

(i) $$(a^2 b^3 c^4)^{\frac{1}{2}} / ((bc^{\frac{3}{2}})/(ac^{\frac{1}{2}})^3),$$

(ii) $$\frac{1}{2} + \frac{39}{51} - \frac{9}{34},$$

(iii) $$\frac{2-\sqrt{5}}{\sqrt{5}+1},$$

[5 marks]

3. Given $y = x^2 - 10x + 35$, complete the square and deduce the minimum value of y. [3 marks]

4. Using $\sin(A+B) = \sin A \cos B + \cos A \sin B$, evaluate $\sin\left(\frac{7}{12}\pi\right)$, leaving your answer in surd form. [3 marks]

5 (a). Find $\sum_{i=10}^{20}(2i-1)$. [3 marks]

(b). Without using your calculator, show that

$$16 + 8 + 4 + \cdots + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = 31.9375\ .$$

[3 marks]

6 (a). On the same graph, sketch $y = 3/(x-2)$ and $y = x$. Find the values of x for which $x > 3/(x-2)$.

(b). Sketch $3\sin(4\theta)$ for $-\pi \leq \theta \leq \pi$. [6 marks]

7. Solve the equations $7a - 2b = 3$, $2a + 3b = 8$. [5 marks]

8. Given that $f(x) = (2x+1)/(x-3)$, find $f^{-1}(x)$. [4 marks]

9. Find the first and second order derivatives of $y = x^2\ \exp(2x)$ with respect to x. [4 marks]

10. Evaluate the integral

$$\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \cos(3\theta)\mathrm{d}\theta.$$

[4 marks]

11. Show that $u = A\sin(x - ct)$, where A and c are constants, satisfies the one dimensional wave equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}.$$

[3 marks]

12. Simplify $(1 + 2j)^4$. [4 marks]

13. Simplify

$$\left|\frac{1}{2+3j} + \frac{1}{3-2j}\right|.$$

[5 marks]

SECTION B

14. The hyperbolic functions cosh and sinh are defined by

$$\cosh(x) = \frac{1}{2}(e^x + e^{-x}), \qquad \sinh(x) = \frac{1}{2}(e^x - e^{-x}).$$

Find the derivatives of these functions, expressing your answer in terms of hyperbolic functions.
Show that

$$\cosh^2(x) - \sinh^2(x) = 1,$$

$$\cosh^2(x) + \sinh^2(x) = \cosh(2x),$$

$$2\cosh(x)\sinh(x) = \sinh(2x).$$

Sketch $\cosh(x)$ and $\sinh(x)$.

[15 marks]

15 (a). Evaluate the integral

$$\int x^2 \mathrm{e}^{2x}\ \mathrm{d}x.$$

[7 marks]

(b). Use partial fractions to evaluate the integral

$$\int_4^5 \frac{12}{x^2 - 9}\ \mathrm{d}x,$$

leaving your answer as a single logarithmic term. [8 marks]

16. Find the coordinates of the stationary points of

$$y = f(x) = (x-2)(x-3)(x+1).$$

Find the equation of the tangent to this curve at $x = 0$.
Find the co-ordinates of the point where this tangent crosses the curve.
Find the co-ordinates of the point of inflection.
Sketch $f(x)$. [15 marks]

17 (a). Given that $z = \cos\theta + j\sin\theta$, express $1/z$ as a complex number.
Expand and simplify

$$\left(z + \frac{1}{z}\right)^5.$$

Hence show that

$$\cos^5\theta = \frac{1}{16}(\cos 5\theta + 5\cos 3\theta + 10\cos\theta).$$

[9 marks]

(b). Express $4\sqrt{2}(1-j)$ in polar form.
Hence, find the 3 roots of the equation $z^3 = 4\sqrt{2}(1-j)$,
giving your answer in polar form.

[6 marks]

18 (a). Evaluate the double integral

$$\int\int_{\mathcal{R}} (x-y)\mathrm{d}x\mathrm{d}y$$

over the region $\mathcal{R}$ $(0 \le x \le a,\ 0 \le y \le \sqrt{a^2 - x^2})$.
Sketch the region $\mathcal{R}$. [10 marks]

(b). Find all the second order derivatives of $w = (x^2 + y^2)^n$.
Simplify

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}.$$

[5 marks]