2MM15

Instructions to candidates

Candidates should answer the WHOLE of Section A and THREE questions from Section B. Section A carries 55% of the available marks.

SECTION A

1 (a). Express $(63)^5$ in standard form, accurate to 4 significant figures.

(b). Simplify

$$\frac{(a^{3/2}b^{1/2}c^2)^4}{(ab^{-1})^2(bc^{-1})^{-2}}.$$

(c). Evaluate $\binom{8}{3}$. Hence, write down the value of $\binom{8}{5}$.

(d). Evaluate $\sin^{-1}(0.623)$ in radians to 3 decimal places.

[9 marks]

2 (a). Find $\sum_{i=1}^{25}(2i)$.
Hence, show that the sum of the first 25 odd numbers is 625. [4 marks]

(b). Write down the value of the sum of the infinite series

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots.$$

[2 marks]

3 (a). Sketch

$$y = 1 + \frac{1}{x-1}.$$

(b). Sketch $2\sin(2\theta)$ for $-2\pi \le \theta \le 2\pi$. [7 marks]

4. Solve the equations $3x - 2y = 9$, $2x - 3y = 1$. [6 marks]

5. Find the roots of the equation $6x^2 - 11x - 10 = 0$. [5 marks]

6. Differentiate $y = x^2(\sin 3x)$ with respect to x. [3 marks]

7. Evaluate the integral

$$\int_0^{\frac{\pi}{12}} \sin(4\theta)\mathrm{d}\theta.$$

[5 marks]

8. Given $\phi(x,y) = \ln(x^2+y^2)$, find $\frac{\partial \phi}{\partial x}$ and $\frac{\partial^2 \phi}{\partial x^2}$.

Without further differentiation, write down $\frac{\partial^2 \phi}{\partial y^2}$.

Hence show that

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0.$$

[7 marks]

9. Given that $z_1 = 4+3i$ and $z_2 = 3+4i$, find $|z_1|$, $|z_2|$ and $|z_1+z_2|$.

Verify that $|z_1| + |z_2| \neq |z_1+z_2|$.

Simplify z_1/z_2. [7 marks]

SECTION B

10 (a). Expand $(x+\delta x)^5$.

Hence, find the derivative of $y = x^5$ from first principles. [8 marks]

(b). Find the first and the second derivatives of $I = \mathrm{e}^{-t}\sin 2t$ with respect to t.

Hence, show that

$$\frac{\mathrm{d}^2 I}{\mathrm{d}t^2} + 2\frac{\mathrm{d}I}{\mathrm{d}t} + 5I = 0.$$

[7 marks]

11. Evaluate the integrals

(i)

$$\int_0^\pi x^2 \cos x \; \mathrm{d}x,$$

(ii)

$$\int_0^2 \frac{x}{x^2+1} \; \mathrm{d}x.$$

[15 marks]

12. Given $y = f(x) = x^3 - 5x^2 - 2x + 24$, evaluate f(1), f(2) and f(3).

Hence write down one factor of f(x).

Divide f(x) by this factor to obtain a quadratic factor.

Factorise this quadratic function.

Differentiate f(x) with respect to x. Find the values of x for which this derivative is zero.

Find the value of x for which $\frac{\mathrm{d}^2 f}{\mathrm{d}x^2} = 0$.

Sketch f(x). [15 marks]

13 (a). Use De Moivre's theorem to show that

$$\cos 4\theta = 1 - 8\cos^2\theta + 8\cos^4\theta.$$

[8 marks]

(b). Write down the value of $\mathrm{e}^{2ik\pi}$, where k is an integer.

Find the complex numbers z for which $z^3 = 1$. Give your answers in the form $re^{i\theta}$ and the form $x + iy$.

Show these numbers as points on the Argand diagram. [7 marks]

14 (a). Evaluate the double integral

$$\int\int_{\mathcal{R}} \rho(x, y)\mathrm{d}x\mathrm{d}y$$

over the region $\mathcal{R}$ $(0 \le x \le y^2,\ 0 \le y \le 1)$ when $\rho(x, y) = x^2 + y^2$.

Give one physical example for a double integral. [9 marks]

(b). Express the cartesian coordinates x and y in terms of the plane polar coordinates r and θ.

Find $\frac{\partial x}{\partial r}$ and $\frac{\partial y}{\partial \theta}$.

Express r in terms of x and y. Find $\frac{\partial r}{\partial x}$. Deduce that

$$\frac{\partial x}{\partial r} \ne \frac{1}{\frac{\partial r}{\partial x}}.$$

[6 marks]