SECTION A

1. Let U be the set $\{0,1,2,3,4,5,6,7,8,9\}$ of digits, E the subset of even digits, $P=\{2,3,5,7\}$, $T=\{3,6,9\}$. List the elements of each of the sets $E\cap T$, $P\cup E$, $E-(T\cup P)$, $E\cap P\cap T$, $U-(E\cup P\cup T)$.

[5 marks]

2. Write down the number of strings of length 5 in the 5 letters a,b,c,d,e. Say how many of these strings

(i) contain the letter c just once;

(ii) start with the letters bc;

(iii) satisfy both (i) and (ii).

[7 marks]

3. In how many ways can a club with 21 members elect

(i) a chairman, secretary and treasurer (all different);

(ii) 3 further members of the committee?

[6 marks]

4. Write out rows 0 to 6 of Pascal's triangle. Expand $(1-x^2)^5$ and $(2x-3)^4$ in powers of x.

[7 marks]

5. Calculate truth tables for the following expressions:

(i) $(P\wedge Q)\vee(\overline{P}\wedge Q)$;

(ii) $(P\rightarrow Q)\vee(P\rightarrow\overline{Q})$;

(iii) $(Q\rightarrow P)\vee(\overline{Q}\rightarrow P)$.

Decide which, if any, is a tautology or contradiction. [7 marks]

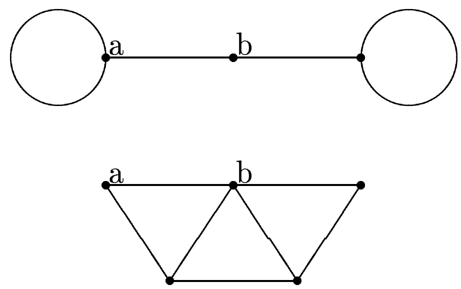
6. Decide whether or not the following arguments are valid, and give reasons for your decision:

(i) $P\vee Q,\ \overline{P}\vee R\vdash Q\wedge R$,

(ii) $P\vee\overline{Q},\ P\rightarrow R\vdash Q\rightarrow R$.

[8 marks]

7. For each of the graphs G_1, G_2 shown, decide



(i) the valencies of the vertices labelled a, b;

(ii) whether or not the graph is simple;

(iii) whether or not it is possible to remove one edge so that the resulting graph is disconnected.

[6 marks]

8. Let A denote the set of strings of length 2 in the alphabet $\{1, 2, 3, 4\}$; R the relation on A consisting of those pairs (ab, cd) with $a \leq c$ and $b \geq d$. Show that R is a partial order and draw its Hasse diagram.

[9 marks]

SECTION B

9. (a) Quality control in a factory rejects 43 parts from an assembly line. Of these, 28 had a paint defect, 17 had a packaging defect, 13 had an electronics defect, 6 had both paint and packaging defects, 7 had both packaging and electronics defects and 3 had both paint and electronics defects. How many had all three types of defect?

(b) How many ways are there of distributing 20 (identical) sweets between 4 children, assuming that each gets at least one? [15 marks]

10.(a) Prove by induction on n that for $n \geq 1$,

$$\sum_{k=1}^{n} k(k+1) = \frac{n(n+1)(n+2)}{3}.$$

(b) Obtain the disjunctive normal forms of

(i) $(Q \to P) \wedge \overline{(Q \to R)}$,

(ii) $(R \to P) \wedge \overline{(Q \vee R)}$. [15 marks]

11.(a) The variables are natural numbers 1,2,3, …; the statement $e(n)$ means that n is even; $d(n)$ means that $n = 5$; $s(n)$ means that $n < 10$. Decide, giving reasons, which of the following are true:

(i) $(\exists n)(e(n) \wedge s(n))$,

(ii) $(\exists n)(e(n) \wedge d(n))$,

(iii) $(\forall n)(d(n) \to s(n))$.

(b) Using predicate logic, prove that the following argument is valid.

Every member of the board comes from industry or government. Every member of the government is in favour of staff cuts. John is a member of the board who does not come from industry. Therefore John is in favour of staff cuts. [15 marks]

12. Let Γ be a simple graph; let N_d be the number of vertices of valency d for each d. Show that the numbers V of vertices and E of edges are given by $V = \sum N_d$ and $2E = \sum dN_d$.

Draw a simple graph with $N_1 = 8$, $N_4 = 4$ and $N_d = 0$ for other values of d. Find all spanning trees of your graph. [15 marks]

13.(i) The set $A = \{1, 2, 3, 4\}$ is partitioned into the subsets $\{1, 3\}$ and $\{2, 4\}$. List the ordered pairs in the correspnding equivalence relation.

(ii) A relation R is defined on the set $B = A \times A$ (where A is as in (i)) by $(a, b)R(c, d)$ if $a - b = c - d$. Show that this is an equivalence relation, and list the equivalence classes. [15 marks]