PAPER CODE NO. COMP304	EXAMINER : **Dr U Hustadt and Dr SD Parsons** DEPARTMENT : Computer Science Tel. No. 46702/46760

THE UNIVERSITY
of LIVERPOOL

JANUARY 2002 EXAMINATIONS

Bachelor of Arts : Year 3
Bachelor of Engineering : Year 3
Bachelor of Science : Year 3

KNOWLEDGE REPRESENTATION AND REASONING

TIME ALLOWED : Two Hours and a half

INSTRUCTIONS TO CANDIDATES

Answer **FOUR** questions only

If you attempt to answer more than the required number of questions (in any section), the marks awarded for the excess questions will be discarded (starting with your lowest mark).

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1. (a) What is Bayes' rule? (4 marks)

   (b) Looking at past results for students doing the Knowledge Representation course (KR), I can see that:
   - A student who previously passed Programming Languages (PL) has a 0.4 chance of passing KR; and
   - A student who previously passed Computational Complexity (CC) has a 0.6 chance of passing KR; and
   - A student who previously passed Artificial Intelligence (AI) has a 0.9 chance of passing KR.

   Every student who takes KR, previously has to take one and only one of PL, CC and AI. Of the 100 students for whom I have information, 60 took PL, 10 took CC and 30 took AI.

   (i) What is the prior probability that a student taking KR will pass? (4 marks)

   (ii) If I know that a student has passed KR, what is the posterior probability that the student passed PL? (4 marks)

   (iii) If I know that a student passed KR, what is the posterior probability that the student passed either CC or PL? (4 marks)

   (iv) If I know that a student passed KR, what is the posterior probability that the student passed both CC and AI? (4 marks)

   (c) A random variable W has three possible values, -2, 5 and 10. The probability that W takes these values are 0.6, 0.3 and 0.1 respectively. What is the expected value of W? (5 marks)

2. Let the Kripke model $\mathcal{M} = ((W, R), I)$ be given by

$$W = \{1, 2, 3, 4\}$$
$$R = \{(1, 2), (1, 3), (2, 4), (3, 4)\}$$
$$I = \{(p, \{1, 4\}), (q, \{2, 4\})\}$$

   (a) Depict the labelled directed graph corresponding to $\mathcal{M}$. (4 marks)

   (b) State how we can check whether an arbitrary modal formula φ is true at a world w in a Kripke model $\mathcal{M}$. (5 marks)

   (c) Give formal derivations which determine at which worlds in the Kripke model $\mathcal{M}$ defined above the formula $q \rightarrow p$ is true. (4 marks)

   (d) Give a formal derivation which determines whether the formula $p \rightarrow \Diamond(q \wedge p)$ is true at world 1 in the Kripke model $\mathcal{M}$ defined above. (12 marks)

3. (a) What is a Bayes' decision? (2 marks)

   (b) What do EMV and MEU stand for? (4 marks)

   (c) I have just bought a new house and need to buy a bookcase to hold my big collection of books on decision theory. One Saturday I have the option of either going to the local department store, Swisel, or to the big furniture store IDEKA, or staying at home. If I stay at home I won't get a bookcase, so the utility of this option is 0. If I go to Swisel, then I might get a really good bookcase, utility 40, an adequate bookcase, utility 20, or no bookcase, utility -10 (because of the cost in time of going to the store). If I go to IDEKA, then I can get a reasonable bookcase, utility 30, or no bookcase, utility -20 (because of the long drive to get there).

   Based on my previous experience with Swisel, I know that the probability of getting a good bookcase from Swisel is 0.5, the probability of getting an adequate bookcase is 0.3 and the probability of getting no bookcase is 0.2. Similarly, if I go to IDEKA, then the probabilty of getting a bookcase is 0.9 and the probability of coming away empty-handed is 0.1.

   Draw a decision tree for my problem and solve it to calculate my Bayes' decision. (15 marks)

   (d) What is my minimax decision? (4 marks)

4. Let the knowledge base Γ be given by the following set of assertional and terminological sentences.

$$\begin{aligned} \text{Male} &\doteq \neg\text{Female} \\ \text{Mother} &\doteq \text{Female} \sqcap \exists\text{child.Human} \\ \text{Father} &\doteq \text{Male} \sqcap \exists\text{child.Human} \\ \text{Parent} &\doteq \text{Mother} \sqcup \text{Father} \end{aligned}$$

$$\begin{aligned} (\text{liz}, \text{andy}) &: \text{child} \\ \text{andy} &: \text{Human} \\ \text{liz} &: \text{Female} \\ \text{will} &: \text{Human} \end{aligned}$$

   (a) Give the expanded TBox of the knowledge base Γ. (5 marks)

   (b) Give a formal derivation of the negation normal form of $\neg\text{Mother}$ with respect to the TBox of Γ. (4 marks)

   (c) State the completion rules for the operators $\sqcap$, $\sqcup$, and $\forall$. (6 marks)

   (d) Give a formal derivation which determines whether liz is an element of the concept Parent. (10 marks)

5. (a) With reference to the two-person zero-sum game:

$$A = \begin{bmatrix} -3 & -3 & -2 \\ 1 & -2 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

in which the payoffs are those of the player who chooses the row,

(i) Explain the terms *pure strategy* and *mixed strategy*. (4 marks)

(ii) What is a stable pure strategy solution to the game? (4 marks)

(b) For the two-person zero-sum game:

$$A = \begin{bmatrix} 5 & -2 \\ 1 & 3 \end{bmatrix}$$

in which the payoffs are those of the player who chooses the row, how would you calculate a stable mixed strategy? (9 marks)

(a) With reference to the two-person general-sum game:

$$(A, B) = \begin{bmatrix} (1,1) & (0,6) \\ (6,0) & (5,5) \end{bmatrix}$$

in which A chooses the row and B chooses the column,

(i) What is the Nash equilibrium solution for pure strategies? (4 marks)

(ii) What is the Pareto optimal solution? (4 marks)