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THE UNIVERSITY *of* LIVERPOOL

JANUARY 2005 EXAMINATIONS

Bachelor of Arts : Year 3
Bachelor of Engineering : Year 3
Bachelor of Science : Year 3

Knowledge Representation and Reasoning

TIME ALLOWED : Two Hours and a half

INSTRUCTIONS TO CANDIDATES

Answer **four** questions only.

If you attempt to answer more questions than the required number of questions (in any section), the marks awarded for the excess questions will be discarded (starting with your lowest mark).

1. (Knowledge Representation and Formalisms)

   (a) Explain why formal tools, and in particular logics, are useful for knowledge representation and reasoning. Refer to the three parts that make up a logic. (6 marks)

   (b) Give an example of a sentence in natural language that is ambiguous, and use a logical language to disambiguate. (6 marks)

   (c) State the so-called *knowledge principle* and give a critical account of it; discuss at least three critical points. (13 marks)

2. (Modal Logic)
   Let the Kripke model $M = \langle W, R, I \rangle$ be given by

$$W = \{1, 2, 3, 4\}$$
$$R = \{(1,2), (1,3), (2,2), (3,4)\}$$
$$I = \{(p, \{2, 3, 4\})\}$$

   (a) Draw the labelled graph corresponding to M, that is, draw the Kripke model M. (5 marks)

   (b) Give formal derivations which determine whether the following are true:

   (i) $M, 1 \models \Box p \rightarrow p$

   (ii) $M, 1 \models \Box \Diamond p$ (12 marks)

   (c) Argue whether $M, 1 \models \Diamond(p \rightarrow \Box \bot)$ (8 marks)

3. (Description Logic)
   Let the knowledge base Γ be given by the following set of assertional and terminological sentences.

$$\begin{array}{rlcr} \text{Male} & \doteq \neg\text{Female} & \quad & (\text{liz}, \text{andy}) : \text{child} \\ \text{Mother} & \doteq \text{Female} \sqcap \exists\text{child.Human} & & \text{andy} : \text{Human} \\ \text{Father} & \doteq \text{Male} \sqcap \exists\text{child.Human} & & \text{liz} : \text{Female} \\ \text{Parent} & \doteq \text{Mother} \sqcup \text{Father} & & \text{will} : \text{Human} \end{array}$$

   (a) Give the expanded TBox of the knowledge base Γ. (5 marks)

   (b) Give a formal derivation of the negation normal form of $\neg$Mother with respect to the TBox of Γ. (4 marks)

   (c) State the completion rules for the operators $\sqcap$, $\sqcup$, and $\forall$. (6 marks)

   (d) Give a formal derivation which determines whether liz is an element of the concept Parent. (10 marks)

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4. Epistemic Logic: Semantics
   Consider a distributed system for three processors; the worlds in the Kripke model are thus all vectors $\mathbf{s} = (s_1, s_2, s_3), \mathbf{t} = (t_1, t_2, t_3)$, where s_i and t_i denote the local state of processor i, and where for every $i \in \{1, 2, 3\}$ the accessibility relation R_i is defined by $R_i(\mathbf{s}, \mathbf{t}) \Leftrightarrow s_i = t_i$.
   Suppose for the example that each processor is characterised by only one variable: t, v, k, respectively. Hence, we can denote the worlds as triples of variables (t, v, k). The variable t denotes time, v 'flag' and k colour. The variable t is a natural number ≥ 1, $v \in \{0, 1\}$ and $k \in \{\mathrm{g}, \mathrm{o}, \mathrm{r}\}$. Examples of primitive propositions in this context include $t = 7$, $v = 1$, $k = \mathrm{r}$, $t > 13$, etc.

   (a) Draw a picture of this distributed system, clearly denoting the accessibility relations. (10 marks)

   (b) Give formulas φ, α, and β such that β is not a tautology and

   $$M, (1, 0, g) \models K_1\varphi \wedge \neg E\varphi \wedge E\alpha \wedge \neg C\alpha \wedge C\beta$$

   (15 marks)

5. (Epistemic Logic) We consider the logic $\mathbf{S5}_2$, the standard epistemic logic for 2 agents. Recall that $M_2\varphi$ is shorthand for $\neg K_2\neg\varphi$.

   (a) Which statement, (b) or (c) below, is true? How is such a statement in general proven and how disproven? (5 marks)

   (b) Prove or disprove:
   $\mathbf{S5}_2 \vdash K_1 p \rightarrow K_2 K_1 p$ (10 marks)

   (c) Prove or disprove:
   $\mathbf{S5}_2 \vdash K_1 p \rightarrow M_2 K_1 p$ (10 marks)