PAPER CODE NO. COMP202	EXAMINER : **Dr PE Dunne** DEPARTMENT : Computer Science Tel. No. 43521

THE UNIVERSITY
of LIVERPOOL

SUMMER 2002 EXAMINATIONS

Bachelor of Arts : Year 2
Bachelor of Science : Year 2

COMPLECITY OF ALGORITHMS

TIME ALLOWED : Two Hours

INSTRUCTIONS TO CANDIDATES

SECTION 1: Answer ***all*** questions
SECTION 2: Answer any ***two*** question

If you attempt to answer more than the required number of questions (in any section), the marks awarded for the excess questions will be discarded (starting with your lowest mark).

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Section 1

Answer **all** questions in this section.

1. Define what is meant by the following terms,

(a) An n-vertex **undirected** graph. (**3 marks**)

(b) A **strongly connected directed** graph. (**3 marks**)

(c) The **Time Complexity** of a decision problem. (**3 marks**)

(d) The **complexity class** Polynomial Time. (**3 marks**)

(e) NP-complete decision problem. (**3 marks**)

2. For the 4-vectors,

$$\mathbf{a} = [\, a_0, a_1, a_2, a_3 \,] = [\, 1, 5, 3, 6 \,]$$
$$\mathbf{b} = [\, b_0, b_1, b_2, b_3 \,] = [\, 2, 4, 0, 7 \,]$$

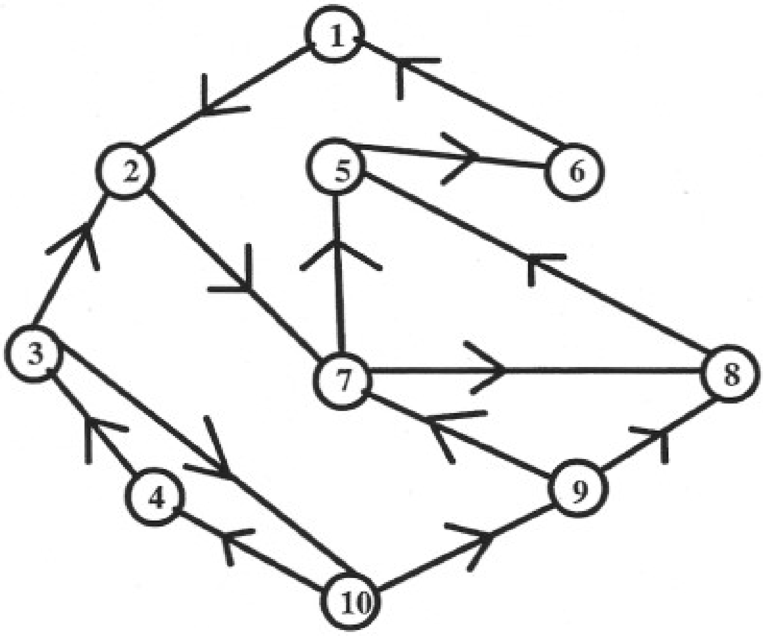
(a) Give the **formal polynomials** $P_{\mathbf{a}}(x)$ and $P_{\mathbf{b}}(x)$ represented by **a** and **b**. (**5 marks**)

(b) Find the **convolution** - $\mathbf{a} \otimes \mathbf{b}$ -

$$\mathbf{c} = [\, c_0, c_1, c_2, c_3, c_4, c_5, c_6, c_7 \,]$$

of **a** and **b**. (**10 marks**)

3. In the 10-vertex **directed** graph,



State whether or not each of the following sets of vertices are in the **same** strongly-connected component and give a **brief** justification for your answer in each case.

(a) Vertices **1** and **7**. (**5 marks**).

(b) Vertices **3** and **4**. (**4 marks**).

(c) Vertices **8, 9,** and **10** (**6 marks**).

4. Carefully define each of the following *NP*-complete decision problems, giving the form that an **instance** of each problem takes **and** the **question** being asked of these instances.

(a) The **Satisfiability Problem** (*SAT*). (**5 marks**)

(b) The **3 Colouring Problem** ($3-COL$). (**5 marks**)

(c) The **Hamiltonian Cycle Problem** (*HC*). (**5 marks**).

Section 2

Answer **two** of Questions 5, 6, and 7.

5.

(a) Briefly outline how the **Discrete Fourier Transform** (*DFT*) can be used to compute the **convolution** of 2 n-vectors,

$$\mathbf{a} = [\, a_0, a_1, \ldots, a_{n-1} \,]$$
$$\mathbf{b} = [\, b_0, b_1, \ldots, b_{n-1} \,]$$

[Do **not** describe the details of the Fourier Transform process itself.]. **(5 marks)**

(b) Carefully describe how the recursive Discrete Fourier Transform method computes $DFT(\mathbf{x}, n, \omega)$ with

$$\mathbf{x} = [\,4, 5, 8, 6\,] \quad ; \quad n = 4 \quad ; \quad \omega = 4$$

and arithmetic calcluations being performed modulo $M = 17$. **(10 marks)**.

(c) The run-time of the Schönhage-Strassen multiplication algorithm is measured in terms of the number of **bit** operations - $O_B()$ it performs..

What is the **principal** difficulty that would arise in attempting to adapt this method to work **directly** with **decimal** (rather than binary) representations? **(5 marks)**

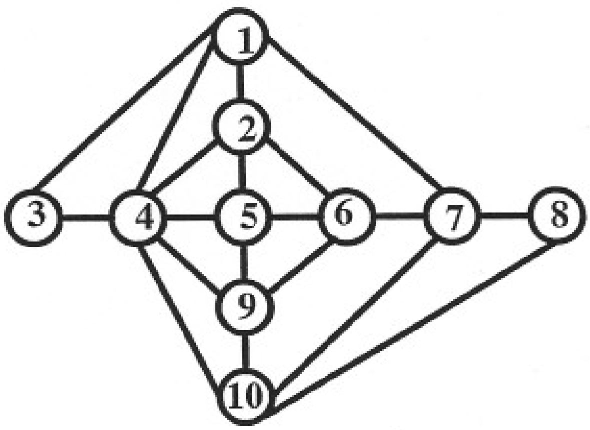
6.

(a) **State** necessary and sufficient conditions for a connected, undirected graph $G(V, E)$ to have an **Eulerian Tour**. **(5 marks)**

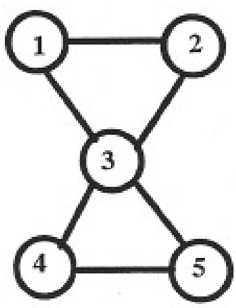
(b) In the undirected graph below, carefully show how the **partial tour**

$$<\{7,6\},\{6,2\},\{2,1\},\{1,7\},\{7,8\},\{8,10\},\{10,7\}>$$

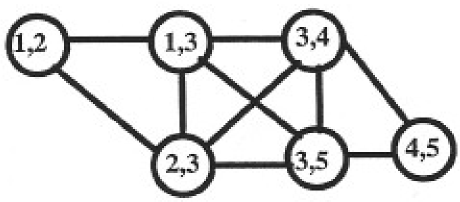
that contains all of the edges adjacent to vertex 7, is extended to a **complete** Eulerian Tour of the graph. **(10 marks)**



(c) The **edge-dual** of a graph $G(V, E)$ is the graph $\hat{G}(W, F)$ formed by replacing each **edge** $\{x, y\}$ in E by a single **vertex** $u_{\{x,y\}}$ in W. The edges of $\hat{G}$ connect those vertices whose corresponding **edges in** E have a common end-point, e.g. as in the example below.



G



$\hat{G}$

G has an Eulerian Tour if and only if $\hat{G}$ has a Hamiltonian cycle. Now consider the following algorithm: given G, construct the graph H for which $\hat{H} = G$ and test if H has an Eulerian Tour. Briefly explain why this algorithm **not** yield a method for deciding if an **arbitrary** graph has a Hamiltonian cycle.
(5 marks)

7.

Given the following instance, F, of $CNF - SAT$,

$$\begin{aligned} & (x_1) \wedge (x_2 \vee \neg x_8) \wedge (\neg x_3 \vee x_4 \vee \neg x_5 \vee x_7) \\ F(X_8) = & \wedge (\neg x_1 \vee \neg x_2 \vee x_4 \vee \neg x_5 \vee x_6 \vee \neg x_7) \\ & \wedge (x_4 \vee x_5 \vee x_7) \wedge (\neg x_4 \vee x_7 \vee x_8) \end{aligned}$$

(a) Identify all of the clauses of F which are not in the form required by the decision problem 3-SAT. **(5 marks)**

(b) **Carefully** show how this instance can be transformed into an instance, G_F, of 3-SAT that is satisfiable if and only if F is satisfiable. **(10 marks)**

(c) An instance of the decision problem $PART-3-SAT$ consists of an instance

$$F(X_n) = \bigwedge_{j=1}^{m} C_j = \bigwedge_{j=1}^{m} (y_{j,1} \vee y_{j,2} \vee y_{j,3})$$

of $3-SAT$ and **two** positive integers r and s (with $r/s < 1$).

The question asked of these being: "is there an assignment to X_n that satisfies **at least** (mr/s) of the clauses C_j?".

Prove that $PART-3-SAT$ is NP-complete, by showing that the restricted case with $r=8$ and $s=9$ is NP-complete.

[Hint: Use a reduction that transforms instances $F(X_n) = \bigwedge_{j=1}^{m} C_j$ of $3-SAT$ to instances $(G_F, 8, 9)$ of $PART-3-SAT$ in which

$$G_F(X_n, Z_t) = F(X_n) \wedge H(Z_t)$$

with $H(Z_t)$ having exactly $8m$ clauses and Z_t is a set of $3m$ new variables.**]** **(5 marks)**

END OF PAPER